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Instructional Practices That Promote Conceptual Understanding and Procedural Fluency

Profile of Practice Brief

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1 Introduction

Introduction

High school graduation rates in the United States are at their highest in U.S. history—81 percent. Even so, nearly one in five students nationally does not graduate from high school and dropout rates are particularly high for students from low-income families, racial and ethnic minority students, and students with disabilities (National Center for Education Statistics, 2015). Though students drop out of high school for a variety of reasons, research consistently reveals that students who fail Algebra I are at an especially high risk (e.g., Oriheula, 2006; Silver, Saunders, & Zarate, 2008). Algebra I, or its equivalent, is typically required for graduation from high school and is a critical gateway course to more advanced mathematics and science courses as well as potential postsecondary degree pursuits (e.g., Ham & Walker, 1999; Helfand, 2006).

Requiring Algebra I of all students does not on its own guarantee that students will succeed in gaining the content knowledge and skills they will need to advance in their education pathways. For example, students who are underprepared for Algebra I may struggle in the course and these challenges could set them on a path toward gradual educational disengagement—not just with mathematics, but with school (Stoelinga, & Lynn, 2013). However, districts and schools can engage in at least five research-based strategies that may promote student success in Algebra I: instructional practices, professional development, instructional coaching, curriculum alignment, and supplementary learning supports for struggling students.[1]

To share information about these strategies, the U.S. Department of Education's High School Graduation Initiative (HSGI) funded the Promoting Student Success in Algebra I (PSSA) project at American Institutes for Research (AIR).[2] The PSSA project is designed to provide actionable information for educational program developers and school and district administrators in three ways. First, a series of research briefs summarizes research on the five strategies above that HSGI grantees are implementing that help struggling students succeed in Algebra I. Second, the project includes a forum for practitioners—district program developers or administrators and teachers—to make connections between the findings from the research briefs and their daily work. The results of these discussions are published in a series of perspective briefs. Third, the project includes profiles of practice that provide an in-depth look at implementation of these five strategies.

[1] For research summaries on each of these strategies, see Sorensen, 2014; Smith, 2014a, 2014b; Walters, 2014a, 2014b.

[2] This brief contains examples of, adaptations of, and links to resources created and maintained by other public and private organizations. This information, gathered in part from practitioners, is provided for the reader's convenience and is included here to offer examples of the many resources and models that educators, parents, advocates, administrators, and other concerned parties may find helpful and use at their discretion. These materials may contain the views and recommendations of various subject matter experts as well as hypertext links, contact addresses, and websites to information created and maintained by other public and private organizations. The opinions expressed in any of these materials do not necessarily reflect the positions or policies of the U.S. Department of Education (Department). The Department does not control or guarantee the accuracy, relevance, timeliness, or completeness of this outside information. Further, the inclusion of links to resources and examples do not reflect their importance, nor is it intended to represent or be an endorsement by the Department of any views expressed, or materials provided.

This profile focused on instructional practices that simultaneously promote procedural fluency and conceptual understanding. As defined by the National Research Council (2001), procedural fluency is "skill in carrying out procedures flexibly, accurately, and efficiently" and conceptual understanding is "comprehension of mathematical concepts, operations, and relations; an integrated and function grasp of mathematical ideas" (p. 5, 118). Figure 1 provides an example of these proficiencies for linear equations, a topic commonly addressed in Algebra I.

Figure 1. Procedural Fluency and Conceptual Understanding Associated With an Algebra I Topic

Students who are procedurally fluent with conceptual understanding are able to not only solve an equation such as $3x + 5 = 20$ for x by performing one or more steps, but they are also able to articulate the mathematics that supports the procedure they used and critique a different approach for solving that same equation. In addition, they are able to conceptualize the solution they find as representing the value for x in the function $f(x) = 3x + 5$ when $f(x)$ is 20. Not only are they able to graph the function $f(x) = 3x + 5$, but also they: (a) understand why it would look like a line, (b) are able to identify the solution to the equation they solved on that line, and (c) know what kind of real-world relationship such a function would model.

Although important components of mathematical proficiency, these competencies have not typically been the focus of instruction. This is particularly true of Algebra I, in which instruction often focuses on manipulation of algebraic symbols with little attention to the underlying concepts (Chazan & Yerushalmy, 2003). To remedy this situation, educators have advocated for a broader focus on procedural fluency and conceptual understanding in Algebra I and mathematics, more generally, and today's college- and career-ready standards emphasize those proficiencies (National Council of Teachers of Mathematics, 1989, 2000, 2006; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Mathematics Advisory Panel, 2008).

The research reviewed by the PSSA project team indicates that instructional approaches that provide students with opportunities to explicitly attend to mathematical concepts (i.e., the meaning behind mathematics procedures, mathematical connections, and the big ideas of mathematics) and struggle with the mathematics (i.e., expend energy to make sense of and reason about mathematics) promote conceptual understanding and procedural fluency (Smith, 2014b). Specific instructional approaches that provide these opportunities for Algebra I students include:

- Explicitly attending to the meaning of algebraic symbols
- Giving students opportunities to reason about algebraic procedures by comparing solution methods
- Challenging students to struggle with algebraic concepts through prediction, investigation, and justification
- Using technology to support mathematical exploration, particularly of algebraic functions
- Engaging students in mathematical modeling activities, including using mathematics to model and solve problems situated in real-world contexts

These practices can be implemented in any classroom, with any set of curricular materials.

The research and focus group data indicate that teachers need professional development, instructional materials, and administrative support to implement the above instructional approaches (Smith, 2014b; Smith, 2016; see http://www2.ed.gov/programs/dropout/instructionalpractices092414.pdf and https://www2.ed.gov/programs/dropout/curricularalignmentperspectivebrief.pdf).

To provide real-world examples of how districts and schools implement and support instructional practices that promote procedural fluency and conceptual understanding in Algebra I, this brief profiles the experiences of rural Harper School District and urban Johnstown Middle School within Green School District.[3] Both sites are committed to supporting student success in Algebra I and do so through an instructional model that emphasizes understanding as well as fluency with algebraic concepts and procedures. The goal of the profiles is not to suggest that other districts mimic the approaches described but rather to share experiences and ideas that other districts may adapt to meet their own communities' needs.

The information presented here emerged from visits to the two sites, during which AIR staff conducted interviews and focus groups with district math leaders, school principals, instructional coaches, teacher leaders, and Algebra I teachers. AIR staff also observed Algebra I instruction in both sites.

[3] See the Appendix for sampling, data collection, and analytic methods, including district selection criteria. All names of people and places in this practice profile are pseudonyms.

2 Lessons From the Field

Lessons From the Field

The following lessons reflect the experiences of the two districts profiled here.

- **Teachers can promote conceptual understanding and procedural fluency in algebra by providing students with opportunities to work collaboratively in making sense of algebraic symbols and procedures through exploration and problem solving**. Although the specific instructional approach used by the two sites differed, both sites focused on sense-making and provided opportunities for students to explore and reason about mathematics in collaborative settings. Harper School District did so through problem-based explorations and problem-solving activities. Johnstown Middle School did so by engaging students in exploration of the connections among algebraic concepts, procedures, and representations.

- **Access to strong professional development and instructional resources is important to supporting implementation of instruction that promotes understanding and fluency.** Both sites offered instructional resources, such as textbooks, sample instructional activities, and access to an instructional math coach, which supported teachers in designing and implementing instruction that emphasized conceptual understanding and procedural fluency. Both sites also provided teachers with professional development that engaged teachers in developing an understanding of the instructional approach, collaboratively planning lessons that implemented this approach, and receiving feedback on instruction. These resources and professional development opportunities were key to supporting the shift to new instructional approaches.

- **A strong instructional math coach can play a role in supporting teachers to make the shift to instruction that promotes fluency and understanding**. Teachers in both sites had access to a strong, innovative math coach who provided instructional resources and professional development to teachers. In Johnstown Middle School, the math coach was school based and worked closely with teachers on a day-to-day basis. In Harper School District, the math coach was at the district level but was still able to provide teachers with supports as needed.

- **Administrators can play a strong role in supporting implementation, particularly if they have opportunities to learn about and provide feedback on the instructional approach.** Administrators in both sites had opportunities to learn about the instructional approach and witness it in action. They then drew upon their learning to support teachers and provide feedback on implementation of those approaches through observation and participation in lesson-planning activities.

3 Harper School District at a Glance

Harper School District at a Glance

Harper School District is located in a rural county in the South Atlantic region. The district serves close to 10,000 students, the majority of whom are from racial and ethnic minority backgrounds; a little over a third of the student population identifies as Hispanic, and a little less than a quarter of the student population identifies as Black or African American. Despite its proximity to highways and easy access to major cities, Harper County is relatively poor, with limited employment opportunities and a declining population. The recent departure of industry and difficulty attracting new industries into the area has caused economic strain on its inhabitants, and approximately 75 percent of the student population qualifies for free or reduced-price lunch. Roughly one half of the adult population has never attained a high school diploma, and student performance on state tests has been quite low. In particular, performance on state tests has been low for Math A, which is the first in a sequence of four high school math courses and covers many of the same topics typically associated with Algebra I. Approximately 50 percent of students are proficient in Math A, as demonstrated by the state Math A test, which is less than the state average of 60 percent proficiency.

Table 1. Harper Schools at a Glance

Geographic Region	South Atlantic
Setting	Rural
District Enrollment	Close to 10,000
Students Eligible for Free or Reduced-Price Lunch	Close to 75 percent
Total Number of Schools	16
Number of High Schools	5
Number of High School Math Teachers	Between 20 and 30
High School Enrollment	Approximately 3,000
Percentage of Students Proficient in Math A	Close to 50 percent
Math A Curriculum	Integrated, commercial textbook series program that emphasizes problem solving, mathematical modeling, and conceptual understanding

Data reported are estimates based on publicly available date for the 2014–15 school year and have been modified minimally to protect the identity of the site.

To better prepare students for life after high school, improve performance on state tests, and increase graduation rates, the district adopted an ambitious instructional model that teachers of all grade levels and content areas are expected to implement. The model emphasizes instruction that promotes deep understanding and provides opportunities for students to actively engage in reading, writing, thinking, and talking in every class, every day. For Math A, implementation of the approach translates to ensuring opportunities for students to work collaboratively to make sense of concepts and procedures through exploration and problem solving, often through activities situated in real-world contexts.

To support implementation of this form of instruction, teachers have access to a number of district-provided resources, including: a district-adopted textbook full of activities that emphasize deep understanding, mathematical exploration, and use of mathematics to model real-world phenomena; a strong, knowledgeable district math coach, who provides additional instructional resources and support to teachers; and professional learning opportunities, including monthly collaborative meetings and periodic workshops, at both the school and district levels. School and district administrators receive professional development on the instructional model and ways of supporting teachers through observation and lesson planning at the district level. They then actively engage in providing teachers feedback and suggestions for continuing to implement the instructional model.

4 Green School District/Johnstown Middle School at a Glance

Green School District/Johnstown Middle School at a Glance

Set in an urban, wealthy county outside of a major city, Green School District could not be more different than Harper School District. The district serves a diverse population of more than 150,000 students. Roughly a third of the student population identifies as White, a third of the student population identifies as Hispanic/Latino, and a quarter of the student population identifies as Black or African American; approximately a third of the student population is eligible for free or reduced-price lunch. Guided by a team of innovative math experts, the district has a long history of supporting math instruction that promotes active student engagement in developing conceptual understanding and procedural fluency. Student performance on mathematics state tests, including Algebra I, has been high.

Table 2. Green School District/Johnstown Middle School at a Glance

Geographic Region	Mid-Atlantic
Setting	Urban
District Enrollment	More than 150,000
Students Eligible for Free or Reduced-Price Lunch	Approximately a third of students
Percentage of Students Proficient in Grade 8 Mathematics	Close to 70 percent
Total Number of Schools	Approximately 200
Number of Middle Schools	20+
Johnstown Middle School Mathematics Teachers	Approximately 10
Johnstown Middle School Enrollment	Approximately 1,000
Johnstown Middle School Students Eligible for Free or Reduced-Price Lunch	More than 60 percent
Johnstown Middle School, Percentage of Students Proficient in Grade 8 Mathematics	Close to 40 percent
Grade 8 and Algebra I Curriculum	District-created

Data reported are estimates based on publicly available date for the 2014–15 school year and have been modified minimally to protect the identity of the site.

It is within this heavily resourced, high-performing school district that Johnstown Middle School, the focus of this profile, is located. In many ways, Johnstown Middle School is similar to schools found in Harper County. The school serves approximately 1,000 students, close to half of whom identify as Hispanic and a third of whom identify as Black or African American. More than 60 percent of Johnstown students are eligible for free or reduced-price lunch, and scores on state tests have been low. As in Harper School District, educational leaders at Johnstown Middle School were interested in improving student performance and looked for ways to change instruction to support student success.

The school adopted an instructional approach that emphasizes connections among mathematics concepts, procedures, and representations through collaborative learning environments. Students, for example, may work

in small groups to understand the relationship between an algebraic equation and its graph. As students connect concepts, procedures, and representations, they begin to make sense of mathematics and develop understanding and fluency. At the same time, the school adopted an eighth-grade algebra policy, which required all eighth-grade students who had not previously taken Algebra I to enroll in the course. The math teachers at Johnstown Middle School committed to supporting all students, including those students who may not have all of the prerequisite skills, to be successful in Algebra I by providing accommodations and additional resources to support student learning. To support implementation of the instructional approach, the math teachers draw on their collective expertise; an innovative, school-based math instructional coach; and number of instructional resources, including those resources offered by the district.

5 Why Instruction That Promotes Understanding and Fluency

Why Instruction That Promotes Understanding and Fluency

The impetus for adopting instructional approaches that promote understanding and fluency was similar in Harper School District and Johnstown Middle School. Faced with low test scores and a student population that was not prepared to move to the next level, educators at both sites committed to holding their students to high mathematics standards and supporting students in attaining those standards through instruction. Both sites turned to an instructional approach that had proven successful elsewhere in the district, that promoted conceptual understanding and procedural fluency in mathematics, and that aligned to rigorous standards for student learning that were later adopted by their respective states.

In Harper School District, educational leaders were searching for a way to better educate their students, making the students more competitive in job and college markets. Once a thriving area for manufacturing and agriculture, the residents of Harper County watched as businesses left the area and the economy began to decline. At the same time, the education system began to suffer. Student state test scores and the percentage of the adult population who had attained a high school diploma were low. As a result, one principal explained, it was difficult to attract new business to the area and improve the economy:

> Companies won't come here. We have land. We are a corridor for [two major highways], so it is ideal . . . As [company owners] look at where to move nice companies, we're always at the top through phases one and two, but when we get to three, they look at the level of education in that community and how trainable the individuals are in the community, and we fall completely out of the running.

To address this problem, district leaders committed to improving educational rigor. The district decided that, although the capacity to memorize facts and apply procedures to solve traditional, textbook-style problems is important, it is equally important and possibly more important to develop an ability to think deeply and critically; to solve real-world, messy problems; and to communicate reasoning to other individuals. To promote these proficiencies in students, teachers needed to change their instruction in all subjects, including mathematics.

District officials turned to one of the high schools in the district for an instructional model that would promote these proficiencies. Spire High School had experienced success in preparing low-income, first-generation college-attending students for college and career and was using an instructional model that emphasized active student engagement in thinking, reading, writing, and talking. In mathematics, this approach fostered the development of conceptual understanding and procedural fluency. The district decided to implement the model in all schools.

At about the same time, the state adopted challenging state academic content standards in mathematics. These standards emphasized conceptual understanding and procedural fluency, as well as an ability to problem solve, think critically, and communicate one's thinking. The adoption and subsequent implementation of the standards provided further motivation to continue with implementation of the new instructional model in mathematics.

In Johnstown Middle School, teachers and school leaders committed to holding students to high expectations and improving test scores and wanted an instructional approach that would support students in meeting those expectations. Although part of a high-performing school district, Johnstown students were not performing well on state tests. For several years, the scores on state tests were consistently low in several subjects, including mathematics. As district officials looked for a way to improve student learning at Johnstown, they turned to Brown Middle School, another middle school in the district that used to have low test scores and had made gains in student math scores, in part, by implementing an instructional approach that focused on helping students understand "why" and not just the "how" associated with algebraic procedures through exploration and communication of mathematical thinking. This approach promotes procedural fluency and conceptual understanding. In hopes of replicating the success of Brown Middle School, the district hired the principal of that school to be the principal at Johnstown Middle School. The principal, in turn, hired the instructional math coach who had been with her at Brown Middle School to be the instructional math coach.

The principal and math coach brought with them a focus on high expectations for students and an instructional approach that promoted sense-making in mathematics, including Algebra I. Although not a district policy, the school adopted the eighth-grade algebra policy, which required all students to have taken Algebra I prior to or during eighth grade. At the same time, teachers committed to supporting student learning in algebra and all math courses, through an instructional approach that emphasized making sense of algebraic expressions and equations; the reasoning behind mathematical procedures; and the connections among mathematical procedures, concepts, and, where appropriate, in real-world contexts. In adopting the eighth-grade algebra policy, teachers and school leaders knew they would likely have students in Algebra I who lacked what are traditionally considered to be prerequisite skills, such as skill in performing operations with fractions and whole numbers by hand, but as the math coach explained, they believed that these students could still reason about mathematics:

> To [support struggling students in Algebra I], you have to think outside of the normal progression of the math box. Math is one of those areas, and I'm not sure why it has become this, where everyone thinks that if you don't have the basic skills, you can't think beyond that. You can't move into application or you can't reason through mathematics, as if you've lost your reasoning power. Well, that just happens to not be true.

As in Harper School District, state adoption of challenging academic content standards in mathematics occurred at about the same time that Johnstown Middle School decided to implement instruction that promoted fluency and understanding and, thus, provided further motivation for the shift.

6 Features of Instruction That Promote Understanding and Fluency

Features of Instruction That Promote Understanding and Fluency

The instructional approaches used by Math A teachers in Harper School District and Algebra I teachers in Johnstown Middle School are quite similar. Both sites implement approaches that emphasize making sense of algebraic symbols and procedures through exploration, problem solving, and modeling of real-world phenomena. Both sites also emphasize the importance of collaborative group work and opportunities for students to explain the mathematical thinking to further solidify their learning. In addition, both sites use technology during exploration to support, not replace, student learning.

Figure 2. Instruction on algebra in these sites emphasized:

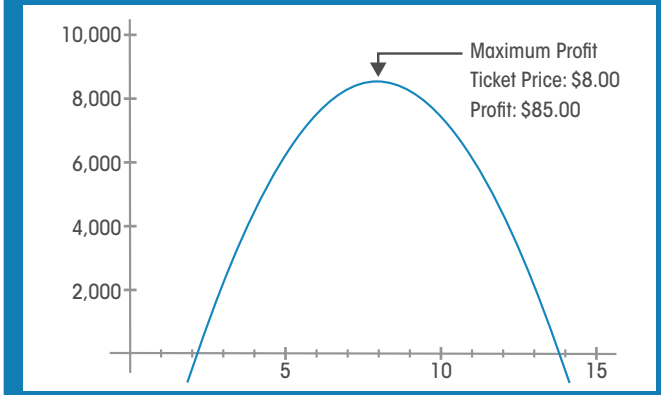
- Making sense of algebraic symbols through exploration, connections, and modeling
- Collaborative group work
- Mathematical explanations and justifications
- Use of technology to support exploration

In Harper School District, instruction is problem based, with opportunities for students to collaboratively explore mathematics, making connections between mathematical representations and to real-world contexts. Math A lessons typically begin with a problem, often situated in a real-world context, which serves as the motivation for an exploration of the new mathematical topic for the day. Students work collaboratively, with teacher guidance, through the exploratory activity to discover a new mathematical idea or procedure and continually explain and justify their thinking with other students. Students often use a graphing calculator, which helps students focus on the development of the mathematical concept, without getting bogged down in calculations. Throughout the exploration, the teacher uses questioning to emphasize the meaning of algebraic symbols and procedures.

In one of the observed lessons, for example, students learned about quadratic functions through an activity in which they were to determine the ticket price that would return the most profit for a music concert. The students began by working collaboratively in small groups to develop a function that modeled revenue as a function of ticket price. They did so by calculating expected revenue when the ticket price was $0, $1, $2, up to $10, looking for a pattern, and constructing the function to represent the pattern. By subtracting the costs associated with the concert from the revenue, students were able to develop a function that described profit as a function of ticket price. Using their graphing calculators, students

Figure 3. Profit Function, Associated Graph, and Solution to the Concert Problem

Profit Function: $f(x) = -250x^2 + 4000x - 7500$



graphed the profit function and identified the ticket price that returned the greatest profit. Figure 3 shows the profit function, associated graph, and the ticket value that returned the greatest profit.

Throughout the lesson, the teacher used questioning to encourage the students to make connections between the fund-raiser context, the symbolic representation of the revenue and profit functions, and the graphs of each. Figure 4 provides examples of questions that the teacher asked the students during the investigation.

Figure 4. Sample Questions Posed to Students During the Concert Problem

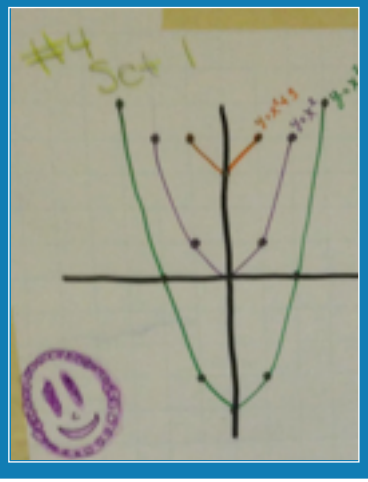
Sample questions posed to students during the concert problem:

- What factors determine the number of tickets sold?
- How does income and profit change as the organizers consider different ticket prices?
- What does "x" represent in the function?
- What do the numbers in the equation represent?
- How can we use the graph to find the ticket price that returns the maximum profit?

Not all Math A lessons in Harper School District are situated in real-world contexts. At times, it is necessary to delve more deeply into the mathematics without the context. Even in these lessons, students explore the mathematics, make connections, and are expected to justify their thinking. For example, in another observed lesson, students were looking for patterns to explain the relationship between the symbolic and graphical representation of a quadratic function, $f(x) = ax^2 + c$. For this lesson, students were given the symbolic representation of three quadratic functions: $f(x) = x^2$, and two other functions that included non-zero values for a and/or c. Students worked collaboratively in small groups to graph the three functions using their graphing calculators, look for patterns, identify a rule to describe the effect of a and/or c on the graph of $f(x) = x^2$, and create a poster to illustrate the rule. Figure 5 shows an example of a student poster illustrating the effect of c on the graph of $f(x) = x^2$.

Figure 5. Student Poster Illustrating the Effect of c on the Graph of $f(x) = x^2$

The graph of $f(x) = x^2$ shifts c units vertically.



The groups took turns sharing the three functions they explored, the patterns they noticed, and the rule they identified with the whole class and answered any questions raised by their classmates. As with the lesson involving the concert tickets, the teacher in this classroom used questioning to encourage students to make sense of the symbolic representations by making connections to the associated graphical representations. Figure 6 shows a sample of the questions the teacher posed to students.

Figure 6. Sample Questions Posed to Students During the Graphing Exploration

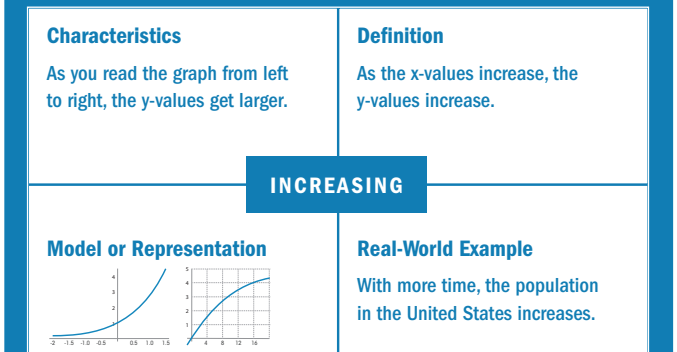
Sample questions posed to students during the graphing exploration include:

- How is $y = ax^2 + c$ related to $y = ax^2$
- How do we see the relationship in the graph? In the table associated with the equation?
- How is c related to the y-intercept [on the graph]?
- What is the difference in the equation that results in the difference in the graph?

In Johnstown Middle School, instruction emphasizes connections among algebraic symbols, representations, and models, through collaborative learning environments. To support student learning, each algebra lesson involves a significant group activity that engages students in developing a strong understanding of the mathematical procedure or concept that is the focus of the lesson and connecting various representations for that new concept or procedure. Often, the teacher models the activity for students, breaks them into small groups, assigns a role to each student in the group, and supports the students as they progress through the activity. The lesson continues with independent work and a summary or closure. Throughout the lesson, there is a strong commitment to providing supports needed for special education students and English language learners to engage in the activities along with other students in the classroom. As in Harper School District, the graphing calculators are often used to support student learning.

Even when the topic does not necessarily lend itself well to student exploration and group work, the Johnstown algebra team, which consists of the algebra teachers, strives to find a way to make it work. This was true of one of the observed lessons, which focused on the development of vocabulary terms used to describe features of algebraic functions. Typically, these lessons begin with the teacher providing the definition followed by student application of the terms. Instead, algebra teachers at Johnstown gave students examples to illustrate the terms and asked students to use the examples to collaboratively come up with definitions and associated representations for those terms.

Figure 7. Example Student Capture Sheet for Vocabulary Lesson



For this activity, teachers arranged students in groups of three and gave the groups two contrasting terms used to classify algebra functions, such as increasing and decreasing. Two of the students in each group played the role of "coach" and were given "playbooks," which included examples to illustrate each of the two vocabulary words. The third student played the role of "player" and was tasked with sorting a set of cards into categories to illustrate the two terms. Using their playbooks, the coaches supported the player in sorting the cards. When the groups had finished sorting the cards, they were asked to complete "capture sheets," one for each term, which served to summarize the activity and provide definitions, examples, and illustrations for each term. Each capture sheet was divided into four parts, as shown in Figure 7. Students drew upon their work in the sorting activity to complete their capture sheet, expressing everything in their own words.

This lesson was observed in three different classrooms. In each classroom, the teacher used different scaffolds to support the particular needs of the students in their classrooms. For example, in one classroom, the teacher used sentence frames to help struggling students write the definition. A sentence frame for the Definition section in the capture sheet illustrated in Figure 7 might read, "As _____ increase, the _____ increase." Students are then required to supply the words to complete the sentence."

7 Supports for Implementation of Instruction That Promote Understanding and Fluency

Supports for Implementation of Instruction That Promote Understanding and Fluency

To support educators in making the shift to algebra instruction that promotes understanding and fluency, instructional leaders in Harper School District and Johnstown Middle School ensured access to professional development and instructional resources for both teachers and administrators. The professional development opportunities engaged teachers and administrators in learning about and experiencing the new approach, in collaborative lesson planning, and in providing feedback on implementation of the approach.

Figure 8. Recommendations from Harper School District and Johnstown Middle School:

- Provide instructional resources that give examples of activities that emphasize fluency and understanding.
- Give teachers and administrators opportunities to learn about and observe the instructional approach in action.
- Provide opportunities for teachers to work collaboratively to discuss and get feedback on their lesson plans and instruction.
- Engage administrators and teachers in observing and providing feedback on lessons, with a lens toward what the student is doing in response to the instruction.
- Identify strong teachers who, if they don't already, will commit to the approach.

In Harper School District, the shift to a new instructional model was supported by a new math textbook series, increased professional learning opportunities, and signing bonuses. In its effort to spread the instructional approach, which emphasized reading, writing, thinking, and talking, that had been successful at Spire High School to other schools in the district, Harper School District adopted the same math textbook series that was used at Spire High School and advocated for its use in all high schools. The textbook series emphasizes problem solving and sense making through explorations, often situated in real-world contexts, and provided a resource for teachers as they switched from lecture-based instruction that focuses on procedures to instruction that promotes active student engagement in developing understanding and fluency. The district also secured funding to enter into a districtwide partnership with an external educational professional development provider that had supported teacher development and implementation of the instructional model at Spire High School. Through this partnership, teachers had access to a range of professional development workshops, including sessions that introduced the new instructional model, provided opportunities to see the instructional model in action, and offered suggestions for how to implement it in the classroom.

Participation in these activities was not required. Similar opportunities were offered to administrators, which helped them be better positioned to support teachers as they implemented the instructional model in their classrooms. Some of the opportunities were provided through two- or three-day workshops. Others were provided as two- or three-hour sessions on a single day. Through the partnership with the professional development provider, Harper School District also received coaching for the principals and, if requested, the teachers at least once a semester. Upon request, the coaches conducted observations, provided feedback, and offered instructional resources for teachers and principals.

To complement the work with the professional education agency, the district also hired a full-time mathematics coach. This innovative, enthusiastic coach had been a math teacher in the district and had experienced success in implementing instruction aligned to the instructional model. The coach provides professional development to teachers, develops pacing guides, and provides a library of materials for teachers to use in their math instruction. These opportunities and resources all emphasize the instructional model and promote procedural fluency and conceptual understanding.

To further support implementation of the instructional model, the external professional development provider recommended lesson tuning and instructional rounds, both of which were instituted in Harper School District. Although the frequency of lesson tuning sessions and instructional rounds varies across schools, most teachers have the opportunity to participate in at least one lesson tuning session and at least one instructional round per month.

Lesson tuning is a technique used to improve lesson plans. Teachers from all content areas and school administrators come together, review a few lesson plans that will be implemented in the upcoming week, and provide suggestions on how to improve the lessons with an eye toward the instructional model. One Math A teacher described the process as follows:

> You're stuck at a point with a lesson and you want to make it beneficial and meaningful to the kids and you just kind of throw it out there. Like I'll say, "This is what I am doing and this is where I want to go," and everybody will come up with ideas and brainstorm. Most of the time you take something from everybody and you make your lesson by pulling those pieces together. The feedback is awesome. I honestly believe I need that English teacher and that social studies teacher with the other side of their brain to give me that . . . We need each other.

Instructional rounds provide opportunities for teachers to observe each other's lessons, often with an administrator, to provide feedback to the teacher on what the students are doing and the extent to which they are engaged in the activities being promoted by the instructional model: reading, writing, thinking, and talking. Instructional rounds may be done in

Figure 9. Sample Questions to Guide Instructional Round Observations

Sample questions to guide observation during instructional rounds include:

- What is the evidence that students are using their roles in the group to dig deeper into the topic?
- What student comments show relevant and meaningful discussions about the text?
- What evidence demonstrates that students are having relevant and meaningful discussions?
- How are students supporting their responses with evidence?

conjunction with lesson tuning so that teachers observe student engagement in the lesson that was tuned but may be done on a different lesson. Teachers use a protocol to guide their observations. Sample questions from that protocol are shown in Figure 9.

By focusing on the instructional model, instructional rounds provide feedback to the teachers, including those who teach algebraic concepts, on how to actively engage students in making sense of the material.

Finally, in recognition of the need for strong teachers to implement this form of instruction, the district offered signing bonuses for teachers to teach math in the district. In this way, the district was able to attract strong teachers who would commit to the instructional model.

In Johnstown Middle School, implementation of the new instructional approach was supported by a full-time, school-based math coach, a schedule that allowed for collaborative planning and professional development, and a committed group of teachers. When the principal came to Johnstown, she requested a full-time, school-based instructional math coach and a schedule that would provide time for teachers to meet in content teams every other day. Both the coach and the schedule were a departure from the norm in the district, but the principal felt that the coach and the schedule were a necessity:

> I needed somebody that could just dedicate themselves 100 percent to really growing teachers and supporting teachers. And, if [the coach is] in the classroom [as a classroom teacher] it's hard to do that because then you're trying to balance the needs of your students with the needs of adults [the teachers]… Central office staff and myself would meet and talk about where we were and look for ways the district could support the school knowing that we had a lot of needs. The district provided [the coach]. I [also] asked for a schedule… that allows for departments to meet every other day. It's very structured with very specific guidelines for what happens at that time.

During the first year, the math coach spent time with teachers, helping them to understand how students' views of themselves and the content can impact their learning. During that time, teachers began to develop an instructional approach and associated lesson activities that would support student success in mathematics. Over time, as teachers implemented the lessons and examined what did and didn't support student learning, the mathematics team refined the approach and the lessons to maximize student understanding and success in mathematics.

The shared planning time that teachers get every other day is a time for teachers to plan lessons and continue to build their understanding of mathematics content and pedagogy. At times, teachers plan together as grade-level or course-specific teams (i.e., Algebra I or geometry). At other times, teachers will meet as a whole department to delve more deeply into the mathematics content through problem solving and cross-grade and cross-course discussions of how that content develops vertically, over time. At still other times, teachers will work together to discuss challenges and help each other brainstorm solutions. Often, the topic of the meetings will align with a topic covered in a more general, schoolwide professional development session. For example, a general session might focus on a topic such as collaborative learning strategies. The math coach will then design the math meeting related to that topic and engage teachers in a discussion of how they can incorporate those collaborative learning strategies into their instruction, while maintaining their emphasis on fluency and understanding through multiple representations.

In addition to her work with the teachers during department meeting times, the math coach observes and gives feedback to teachers about their lessons and provides them with instructional resources to support their instruction. In her observations, the coach focuses on teachers' implementation of the approaches discussed in the extra period. Not only does she pay attention to what the teacher is doing but she also pays attention to what the students are doing during the activity. During small-group time, the coach may even talk with students about their understanding, audio record the discussion, and bring the audio recording to a department meeting to discuss aspects of student thinking with the math teachers. The instructional resources that the math coach provides to teachers are often associated with the National Council of Teachers of Mathematics and, thus, represent the latest thinking on mathematics teaching and learning. In lieu of a textbook, teachers use these materials along with the materials developed by the district to implement instruction.

Figure 10. Example Summary Sheet for a Big Idea in Algebra I

It's all about that BASE!!! Solving Exponential Equations

STEP 1: To solve the equation, rewrite the equation so that the expressions on each side of the equation have the same base.

$27^{2x+5} = 3^{-3}$

$(3^3)^{2x+5} = 3^{-3}$ *27 is equal to* 3^3

$3^{3(2x+5)} = 3^{-3}$

STEP 2: We now have expressions that have the same base. In order for the expressions on each side of the equation to be equal, the exponents need to be equivalent. Write an equation to show the equivalence of the exponents.

$3(2x + 5) = -3$

STEP 3: Solve the new equation for "*x*".

Johnstown also was able to dedicate one special education certified teacher to Algebra I. This algebra intervention teacher co-teaches with the Algebra I teachers and offers suggestions for how to improve instruction, not only for students with special needs but also for English language learners. The inclusion of the sentence frames in the vocabulary lesson, described in the previous section, were at the suggestion of the algebra intervention teacher and were helpful to all learners. In addition to supporting instruction, the algebra intervention teacher develops and provides summary sheets for students (and their parents) that address the big ideas of the course and periodically compiles a set of YouTube videos, in English and Spanish, which address the content being covered in Algebra I. An example of a summary sheet is shown in Figure 10. By clearly identifying and explaining key concepts in Algebra I, these additional resources support struggling students.

Even with the number of resources and supports offered by Johnstown Middle School and the district, the transition would not have been possible without a committed group of teachers. When the principal came to the school, teachers had the option of leaving or re-interviewing for their jobs. A few teachers elected to go to another school and, over time, some of those newly hired teachers have elected to leave. The teachers who have engaged in the process over time, however, are indeed committed to the approach and its impact on their students.

8 Challenges and Facilitating Factors

Challenges and Facilitating Factors

Although both Harper School District and Johnstown Middle School have had success implementing instructional practices that promote fluency and understanding in Math A and Algebra I, implementation was not without challenge. Changing instruction and student expectations can be difficult for teachers as well as for students and their parents. By providing an open, supportive environment, both sites have been able to address those challenges.

Teacher change is difficult and requires teachers who are amenable to change and have support for doing so. Teachers and educational leaders in both Harper School District and Johnstown Middle School identified resistance to change on the part of the teacher as a challenge to implementing instruction that promotes understanding and fluency. Staff in both sites reported access to resources and professional development as key to supporting teachers, including those who are resistant and those who are amenable to change, in making the shift.

At both sites, a strong, innovative math coach provided teachers with instructional materials to use in their classrooms. In Harper School District, the district approved a problem-based, high school mathematics textbook series that was full of activities that emphasized the instructional mode and, within mathematics, conceptual understanding and procedural fluency in all content areas, including Algebra I. In Johnstown Middle School, teachers had access to the district-developed materials, which did much of the same. Access to these resources can help to ease the transition to a new instructional model and support continued implementation.

Possibly more important, teachers in both sites also had opportunities to develop an understanding of the new approach and work collaboratively to plan, observe, and give feedback to each other. In Harper School District, teachers watched the instructional model in action, participated in professional learning activities during which they experienced the new instructional approach (and textbook activities) as students, and had opportunities to plan and observe lessons through lesson tuning and instructional rounds. In Johnstown, teachers had an established time to meet, discuss challenges they faced, think deeply about the mathematics they were going to teach, discuss connections to other grade levels, and collaboratively plan lessons. The Harper School District math coach described the importance of the professional learning as follows:

> When we can get teachers outside of their own classrooms where they can see a set of students that are like their own, if you can get the teachers to see a successful classroom . . . I think it helps them see, okay this can be done. It's going to be work, I'm going to pull my hair out a lot of days, but I'm seeing right now the benefits.

These activities alone may not be enough, however, to support teacher change. In both sites, teachers were there because they were committed to the approach. From the outset, Harper School District leaders used hefty signing bonuses to attract strong teachers, who believed in and could implement instructional approaches that support conceptual understanding and procedural fluency, to transfer to the district. In Johnstown Middle School, math teachers who were not committed to the new instructional approach decided to leave the school.

Students, too, may have difficulty shifting to new expectations, and supports are needed to ensure their success. Both Harper School District and Johnstown Middle School set high expectations for students. Both sites required students to make connections between algebraic procedures, concepts, and representations; to work collaboratively; and to solve non-routine problems. The Johnstown eighth-grade algebra policy took high expectations for students even further because it required all eighth-grade students to enroll in Algebra I, if the students had not already taken it. Not surprisingly, lack of prerequisite skills and initial discomfort participating in a learning environment that requires more than just listening to lectures and taking notes were cited as challenges associated with switching to instruction that focuses on active student engagement in developing fluency and understanding.

To address this challenge, both sites offered students supports, including modification of activities to address student needs and, as the research cited in the PSSA literature review (Smith, 2014b) suggests, allowing students to productively struggle with the material. The Algebra I lead teacher at Johnstown described allowing students to productively struggle as follows:

> Not answering their questions, just walking away because they want you to just stand there and guide them through the work from start to finish and they want to know that they're right . . . They start to realize, they need to figure it out for themselves and that they can figure it out if they would just think about it.

Teachers in both sites also mentioned use of grouping to support student learning. For example, teachers might put English language learners who are high- and low-performing in math together in one group so that they can support each other, both in the mathematics and in the language.

Parents may struggle with the shift to a focus on conceptual understanding as well as procedural fluency, and providing opportunities for them to learn more can ease their concerns. As parents see algebra moving from the rote manipulation of symbols that they experienced as students toward making sense of those symbols and manipulations through investigation, problem solving and modeling, they may begin to wonder if the approach is a good one. This is particularly true if parents see their student struggling. Teachers and administrators in both sites indicated that open communication

with parents can address parents' concerns. In Johnstown, the algebra intervention teacher identifies YouTube videos, in both English and Spanish, which parents can view to better understand the mathematics. In one school in Harper School District, the principal helps parents to understand that the ability to think deeply and critically, problem solve, and communicate your thinking in collaborative groups is important to the workplace, and that is why instruction focuses on those proficiencies. The principal also shows parents data about the types of skills needed for the workforce to suggest that this form of instruction is valuable to student success in later life.

9 Key Considerations for Education Decision Makers

Key Considerations for Education Decision Makers

As decision makers consider how best to implement a practice within their local context, it is important to consider the insights gained from a variety of sources. Table 3 shows the instructional approaches that support conceptual understanding and procedural fluency as identified in the systematic literature review (Smith, 2014b; see https://www2.ed.gov/programs/dropout/instructionalpractices092414.pdf) and the ways in which those practices were reflected in Harper School District and Johnstown Middle School.

Table 3. Instructional Practices That Promote Understanding and Fluency in Algebra and How Those Findings Were Reflected in the Profiled Sites

Instructional practices that promote procedural fluency and conceptual understanding in algebra	How the finding was reflected in Harper School District and Johnstown Middle School
Give explicit attention to the meaning of algebraic symbols.	In both sites, instruction provided opportunities for students to understand connections between and among algebraic concepts, procedures, and representations. In addition, both sites emphasized use of algebra to model real-world phenomena.
Give students opportunities to reason about algebraic procedures by comparing solution methods.	Although both sites implemented instruction that emphasized connections between algebraic procedures and the underlying concepts, they did not necessarily do so through comparing solution methods.
Challenge students to struggle with algebraic concepts through prediction, investigation, and justification.	Both sites emphasized exploration and student communication of mathematical reasoning.
Use technology to support mathematical exploration, particularly of algebraic functions.	Both sites used graphing tools, including the graphing calculator, to support exploration in ways that allowed students to focus on the big ideas without being bogged down with tedious calculations.
Engage students in mathematical modeling activities, including using mathematics to model and solve problems situated in real-world contexts.	Both sites emphasized the connection between mathematics and real-world contexts.

The review of research led to the identification of a list of implications for supporting instruction that promotes conceptual understanding and procedural fluency in Algebra I that education decision makers might consider. The teachers and teacher leaders who participated in the focus groups for the perspective briefs agreed that the supports mentioned were

important to a successful transition and implementation of instruction that promotes understanding and fluency (Smith, 2016, see https://www2.ed.gov/programs/dropout/curricularalignmentperspectivebrief.pdf). Table 4 shows the list of research implications for educational decision makers to consider and how those were reflected in the two profiled sites.

Table 4. Considerations for Educational Decision Makers and How Those Findings Were Reflected in the Profiled Sites

Considerations for education decision makers	How the finding was reflected in Harper School District and Johnstown Middle School
Provide professional development to teachers that supports teachers in implementing instructional practices that promote understanding and fluency.	Both sites offered extensive professional development that provided teachers opportunities to develop an understanding of the approach, plan with other teachers, and receive feedback on the plans and the implementation of those plans.
Use observation protocols that target these instructional practices that promote understanding and fluency and develop ways to provide feedback on the observation guided by those protocols to teachers.	Teachers in both sites were observed by other teachers and by administrators. Observation protocols focused on the extent to which students were engaged in activities that promoted understanding and fluency.
Provide teachers with curricular materials that emphasize instruction that promotes understanding and fluency.	Both sites provided instructional resources to teachers that emphasized instruction to promote understanding and fluency. Some of these resources came from the district and some resources came from an instructional mathematics coach. In Harper School District, the district adopted a textbook that emphasized those proficiencies.
Develop pacing guides that support the use of instruction that promotes understanding and fluency.	Although not presented in this profile, both sites did provide pacing guides to teachers that ensured time spent on both concepts and procedures.
Develop pacing guides that support the use of instruction that promotes understanding and fluency.	Although not presented in this profile, both sites did provide pacing guides to teachers that ensured time spent on both concepts and procedures.

10 Appendix

Appendix. Sampling, Data Collection, and Analytic Methods

In selecting sites for Promoting Student Success in Algebra I, the primary objective was to identify districts and schools that implemented activities associated with the five topical areas that are the focus of this project (instructional practices, professional development, instructional coaching, curricular alignment, and supplementary learning opportunities). In addition, for the practice profiles to be of greatest utility to practitioners and policymakers, American Institutes for Research (AIR) sought to identify sites that were implementing the practices identified in the research (see Research Briefs) as showing the strongest evidence of effectiveness. To enhance the probability that practitioners would identify with the school and district sites, AIR sought variation with regard to urbanicity, school size, and student demographics. Briefly, the selection criteria included:

- Criterion 1: Sites will represent exemplars.
- Criterion 2: Each site should provide some evidence of improved outcomes.
- Criterion 3: Sites will reflect geographic diversity.
- Criterion 4: Sites will reflect the diversity of enrolled students.

With regard to instructional practices that promote conceptual understanding and procedural fluency, specific expectations were as follows:

> Selected sites will be schools with an established track record (at least three years) of teaching algebra for conceptual understanding and procedural fluency. Schools that systematically promote this type of teaching for all algebra teachers are those in which we would expect to see lessons, tasks, and assessments in which concepts and procedures are mutually reinforced. Finally, the project team will seek schools that have some evidence of improved student outcomes (e.g., algebra end-of-course exam scores increased since the school implemented this type of instruction).

Both sites profiled here had an established track record of teaching algebra for conceptual understanding and procedural fluency. Lessons were designed to promote these proficiencies, as were tasks and assessment. In Harper School District, evidence of improved student outcomes could be seen in Spire High School, where instructional practices that promote understanding and fluency had been implemented the longest. The school had successfully prepared 100 percent of its low-income, first-generation college students for college and career and had demonstrated higher math scores than the rest of the county. In Johnstown Middle School, pass rates in Algebra I had increased from approximately 35 percent to approximately 50 percent with implementation of instruction to promote conceptual understanding and procedural fluency.

Teams of at least three project staff visited each of the profiled sites following training in data collection procedures. On-site data collection activities included interviews, focus groups, observations of algebra instruction, and document data collection. The interview and observation protocols were developed by project staff with expertise in algebra content, research on instructional practices, and qualitative research. Each protocol was piloted and refined based on feedback from practitioners before being fielded for this project. A total of 12 educators were interviewed across the two sites: 3 administrators and 2 teachers in Harper, and 4 administrators and 3 teachers in Green. All interviews and focus groups were audio-recorded and transcribed. Following the school and district visits, the project team immediately summarized their observations. These observations were used to identify initial themes and supported the development of codes.

Interview and focus group transcriptions were coded in Dedoose, a qualitative data analysis software package. Prior to coding, the project team developed a set of codes with associated definitions and trained staff for consistent application of codes. Additionally, a senior staff member reviewed the coded data as a quality control procedure. Coded data enabled the project team to retrieve data on common topics across interviews. For example, code retrieval allowed the team to analyze all the relevant data points on such topics such as district supports or teacher collaboration. The coding process ultimately allowed for the identification of prominent themes and informed the development of the Key Considerations.

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