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Instructional Practices to Support Student Success in Algebra I

Research Brief

This research brief is one of five that summarize the literature in different topic areas[1] related to helping struggling students in Grades 6–9 succeed in algebra. The research briefs are part of the *Promoting Student Success in Algebra I* (PSSA) project funded by the U.S. Department of Education's High School Graduation Initiative (HSGI). The PSSA project at American Institutes for Research is designed to provide actionable information for educational program developers/administrators in three ways. First, these research briefs together will summarize research on five strategies being implemented by HSGI grantees that help struggling students succeed in Algebra I, a critical gateway course for high school graduation and enrollment in college. Second, the project includes a forum for practitioners—district curriculum developers/administrators and teachers—to make connections between the findings from the research briefs and their daily work, with the results of these discussions published in a series of perspective briefs. Third, the project includes profiles of practices that provide an in-depth look at implementation of these five strategies.

This research brief focuses on instructional practices. Typically, instruction in algebra focuses on symbolic manipulation and algebraic procedures, with little attention to the connections between these procedures and the underlying mathematical concepts (Chazan & Yerushalmy, 2003). When students are expected to memorize and operate with a set of rules that are seemingly meaningless, they may become frustrated and eventually fail Algebra I. Recommendations for student learning in algebra (and mathematics, more generally) address this concern and emphasize that proficiency in algebra requires more than skill in the rote application of algebraic procedures (National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010; National Mathematics

[1] The five topic areas are Curricular Alignment, Instructional Practices, Supplementary Learning Supports, Professional Development, and Instructional Coaching.

Advisory Panel, 2008; National Council of Teachers of Mathematics, 1989, 2000, 2006). It requires what the National Research Council (NRC; 2001) describes as *procedural fluency and conceptual understanding* (for definitions, see Exhibit 1).

Exhibit 1. Procedural Fluency and Conceptual Understanding

Procedural Fluency – skill in carrying out procedures flexibly, accurately, efficiently

Conceptual Understanding – comprehension of mathematical concepts, operations, and relations; an integrated and functional grasp of mathematical ideas

—NRC (2001, pp. 5, 118)

With the recent implementation of more rigorous College and Career Readiness Standards in mathematics and wide-scale adoption of the Common Core State Standards for Mathematics (CCSSM; NGACBP & CCSSO, 2010), more emphasis is being placed on procedural fluency as well as conceptual understanding than ever before.

What characteristics of instruction might promote these proficiencies in algebra? To answer this question, we reviewed research on instructional practices that support the development of procedural fluency and conceptual understanding in algebra. In particular, we were interested in studies that demonstrated the impact on these aspects of algebraic proficiency, beyond rote application of procedures. The process we used to conduct this review is described in more detail in the Appendix.

Although much of the research in this area does not meet the highest level of rigor described by the What Works Clearinghouse,[2] the research does provide evidence for best practice. Our synthesis of the research indicates that (a) a need exists to reconsider traditional approaches to algebra instruction, (b) instruction that provides the opportunity to make sense of algebraic symbols and procedures promotes procedural fluency and conceptual understanding, and (c) implementing instruction focused on sense making throughout an entire course supports, not inhibits, understanding and fluency. These findings have implications for instructional design, curricular materials, teacher evaluation, and professional development and are described in the Implications section of this brief.

[2] The What Works Clearinghouse was created in 2002 by the Institute of Education Sciences to be a source of information regarding what works in education. See http://ies.ed.gov/ncee/wwc/DocumentSum.aspx?sid=19 for the standards used to evaluate studies.

Synthesis of the Literature

Reconsidering Traditional Approaches to Algebra Instruction

Mathematics instruction in the United States looks very similar: students review previously learned content, the teacher demonstrates new material, and students practice. Often, as demonstrated by the Trends in International Mathematics and Science Study (TIMSS) video studies, the focus of these activities is the application of mathematical procedures (Hiebert et al., 2003; Stigler & Hiebert, 2004). Now, with the introduction of new standards for learning, such as those outlined in the CCSSM (NGACBP & CCSSO, 2010) and the National Council of Teachers of Mathematics standards documents (2000, 2006), students are expected not only to develop skill in applying mathematical procedures but also to develop fluency in working with those procedures and develop an understanding of the underlying mathematics.

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This shift in focus represents a major change, particularly for algebra education. For decades, the heart of algebra education has been learning to manipulate algebraic symbols to solve equations. Now, students are expected to know and be able to do more than solve equations. They are expected to demonstrate fluency in working with algebraic procedures, understand the associated mathematical concepts behind these procedures, and be able to articulate the connections between and among them (Exhibit 2). This presents a challenge for algebra teachers, who must design instruction that promotes not only skill development but also procedural fluency and conceptual understanding.

Exhibit 2. New Standards for Student Learning

> Not only should students be able to solve an equation such as $3x + 5 = 20$ for x by performing one or more steps, but also they should be able to articulate the mathematical principles (e.g., order of operations, properties of equality) that support the procedure they used and critique a different approach for solving that same equation. In addition, they should understand that the solution they find represents the value for x in the function $f(x) = 3x + 5$ when $f(x)$ is 21. Not only should they be able to graph the function $f(x) = 3x + 5$ but also they should (a) understand why it would look like a line, (b) be able to identify the solution to the equation they solved on that line, and (c) know what kind of real-world relationship such a function would model. These are all features of procedural fluency and conceptual understanding.

Research on mathematics teaching suggests that different features of instruction promote the development of skills and understanding. Instruction that is fast paced, characterized by teacher modeling of how to solve problems followed by student practice, and focused solely on getting the correct answer (as opposed to the reasoning behind the answer) promotes the development of mathematical skill, which is only one component of procedural fluency. It does not include an ability to use those skills flexibly and efficiently, which are both components of procedural fluency. In contrast, instruction that (a) explicitly and publically attends to mathematics concepts (i.e., involves students in discussions/explorations that target the meaning behind mathematics procedures, mathematical connections, and the big ideas of mathematics) and (b) provides opportunities for students to struggle with the mathematics (i.e., expend energy to make sense of and reason about mathematics) promotes conceptual understanding. Instruction that promotes conceptual understanding also promotes procedural fluency (Hiebert & Grows, 2007).

These findings suggest that the typical approach to algebra instruction (and mathematics instruction, more generally) may not support the development of the procedural fluency and conceptual understanding demanded by new standards for student learning. Instead of focusing solely on skill in manipulating algebraic symbols, instructional activities should provide students with opportunities to struggle with algebraic concepts and make connections between algebraic procedures and concepts. The challenge is finding ways to do that.

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Making Sense: Promoting Conceptual Understanding and Procedural Fluency in Algebra

A review of recent research that explicitly investigates the impact of specific instructional strategies on procedural fluency and conceptual understanding in algebra identified several strategies that support aspects of both. Common to these practices is an emphasis on sense making. By challenging students to make sense of algebraic symbols and procedures, these strategies combine features of instruction that promote skills with those that promote conceptual understanding and procedural fluency.

Promoting Meaning for Algebraic Symbols

Too often, algebra students are asked to work with algebraic expressions and equations without considering what they might represent. Research indicates that when instruction attaches meaning to the associated algebraic symbols, students develop procedural fluency as well as conceptual understanding. This can be done in a number of ways, including through the use of technology.

One approach to assigning meaning to algebraic symbols involves algebraic expressions (e.g., $3x + 4$). Students are given a list of algebraic expressions and asked to predict whether or not any of the algebraic expressions are equivalent. Some expressions are equivalent but look different (e.g., $2(x + y)$ and $2x + 2y$) and others look similar but are not equivalent (e.g., $2(x – 3)$ and $2x – 3$). After they make their predictions, students are asked to test them by substituting several numerical values for the variables in those expressions. This step emphasizes the meaning of variable. Once they have their results, students are then asked to provide a justification for what they found. Students who experienced this form of instruction on algebraic expressions produced higher pretest-posttest gains on measures of symbolic manipulation and understanding of variable than did students who received conventional, skills-based instruction (Graham & Thomas, 2000).

Another approach for promoting meaning involves mathematical modeling. Students are asked to reason about and work with algebraic equations as a means to model real-world phenomena. This strategy was used with middle-grades students as part of a technology-enhanced unit on proportion and rate,[3] both of which are important topics for the study of algebra. As students moved through the unit on proportion and rate, they used computer-based software to explore and manipulate graphical and symbolic representations of motion. Throughout, they made predictions, compared their predictions with what happened in the real world, and explained differences between their predictions and what they experienced. Students who learned about rate and proportion in this environment performed better on items that assessed skills, the ability to move flexibly among different representations, and conceptual understanding than those who were taught with traditional methods (Roschelle et al., 2007).

Research has also supported the use of prediction and justification within a modeling approach during instruction on linear and exponential relationships (Kasmer & Kim, 2011). In this study, both treatment and control students learned these concepts through instruction that emphasized mathematical modeling and exploration. Instruction for the treatment group, however, was enhanced with additional prediction questions. For example, prior to investigating the solution to a linear system, students were asked, “Do you predict two companies will ever charge the same amount?” (p. 24). Once students made their predictions, they were asked to justify their predictions to others. Students then completed the mathematical exploration, and teachers revisited their predictions at the end of the lesson. Students who experienced the prediction-enhanced instruction demonstrated stronger conceptual understanding and ability to use algebraic symbols to represent relationships than did students in the control group. A later study (Kasmer & Kim, 2012) found that by making predictions, students were able to connect prior knowledge to new mathematics, ultimately supporting them in making sense of and reasoning about algebraic symbols.

[3] The curriculum was developed by the Dana Center at the University of Texas at Austin and the University of Massachusetts, Dartmouth and supplemented with the Sim Calc Mathworlds software.

Reasoning About Algebraic Procedures

As students work with algebraic equations, they are expected to perform symbolic manipulations that become increasingly complex, such as solving the equation $3(x - 1) + 2(x + 2) = 5(x - 1) + 3(x + 2)$ for x. Often, it can seem as if performance on such problems requires merely the memorization of seemingly disconnected rules because students typically learn solution methods one at a time. Research has indicated that asking students to compare and reason about different solutions supports both procedural fluency and aspects of conceptual understanding.

In particular, research has indicated that students who received instruction requiring them to compare and reason about different, correctly worked-out solutions for the same algebraic equation demonstrated greater gains on measures of procedural fluency and aspects of conceptual understanding than students who received instruction that required them to reason about one solution at a time (Rittle-Johnson & Star, 2007). Further, this approach to instruction was found to result in greater gains than instruction that required students to compare and reason about the same solution method applied to different, but structurally equivalent, problems (i.e., $5(x - 1) = 10$ and $6(x + 2) = 12$) and the same solution method applied to different problem types (i.e., $5(x + 1) = 2(x + 1) + 6$ and $5(x + 1) + 2(x + 1) = 14$; Rittle-Johnson & Star, 2009).

Asking students to reason about *incorrectly* worked-out solutions to algebraic equations may also support the development of procedural fluency and conceptual understanding. Research indicated that students who received instruction that asked them to reason about *both* correctly and incorrectly worked-out solutions to algebraic equations performed as well on measures of aspects of procedural fluency and better on measures of aspects of conceptual understanding than students who received instruction that asked them to reason only about *either* correctly or incorrectly worked-out solutions to algebraic equations (Booth, Lange, Koedinger, & Newton, 2013).

Whole-Course Instruction Focused on Sense Making Supports Fluency and Understanding

Although activities that engage students in making sense of algebraic symbols and procedures hold promise for promoting procedural fluency and conceptual understanding, some would argue that spending too much time asking students to engage in these activities on a regular basis throughout the course will result in lowered levels of procedural fluency. Research on curricula that routinely engage students in these activities have indicated that this is not the case.

Perhaps the strongest evidence that whole-course instruction focused on making sense does not detract from, and actually supports, procedural fluency can be found in curriculum comparison studies. Many of these studies investigated the impact of curricula designed to promote instruction aligned with the NCTM (1989, 2000) standards documents. Analyses suggested that these standards-based curricula provided students with more opportunities to make sense of mathematics than did conventional curricula (Stein, Remillard, & Smith, 2007). Syntheses of curricular comparison studies indicated that in mathematics, students taught with these curricula performed better on assessments of conceptual understanding but no worse on assessments of aspects of procedural fluency (Chappell, 2003; Putnam, 2003; Stein et al., 2007; Swafford, 2003), including standardized tests (e.g., Post et al., 2008; Harwell et al., 2007), than students taught with conventional curricula.

Conceptual approaches to algebra instruction demonstrated a greater positive impact (larger effects) on student achievement than procedural approaches.

Research focused specifically on algebra yielded similar results. A recent review of research on algebra instruction (Rakes, Valentine, McGatha, & Ronau, 2010) found that conceptual approaches to algebra instruction demonstrated a greater positive impact (larger effects) on student achievement than procedural approaches. These findings were consistent with those of curricular studies focusing on algebra-specific student outcomes in both middle-grades mathematics (which includes instruction on algebra topics) and high school Algebra I.

Middle Grades: Impact on Understanding and Fluency With Algebraic Concepts

Two studies specifically examined the impact of using whole-course, standards-based curricula on the algebraic skills and understandings covered in middle-grades mathematics. These studies focused on the impact of *Middle-Grades MATH Thematics* (MT; Billstein & Williamson, 1998) and the *Connected Mathematics Project* (CMP; Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997). Both curricula emphasize, among other things, sense making through reasoning, exploration, and application to real-world contexts (Billstein & Williamson, 1998; Ridgway, Zawojewksi, Hoover, & Lambdin, 2003). One study found that MT students outperformed students in comparison classrooms that implemented conventional curricula on measures of conceptual understanding and no differently on measures of aspects of procedural fluency in algebra (Billstein & Williamson, 2003) The other study found that MT and CMP students outperformed students in comparison classrooms on the algebra-specific items on the state-mandated mathematics exam, which assessed both skills and concepts (Reys, Reys, Lappan, Holliday, & Wasman, 2003).

High School: Impact on Understanding and Fluency in Algebra I

At the high school level, two studies investigated the impact of *Cognitive Tutor Algebra I* (CTAI),[4] a standards-based curriculum that emphasizes making sense through modeling and connections between and among mathematical representations and provides opportunities for individualized instruction on algebra-specific student outcomes. One study (Morgan & Ritter, 2002) found that students who received instruction through CTAI outperformed students who received the conventional curriculum on the algebra end-of-course exam.[5] The second study (Pane, Griffin, McCaffrey, & Karam, 2013) matched schools on a variety of school-level variables and then randomly assigned them to CTAI. In that study, students in the CTAI program outperformed students who received the traditional curriculum on the algebra proficiency exam.[6] The developers of the exams used in both studies claimed to have included items to measure aspects of procedural fluency and conceptual understanding, but neither reported separate outcomes for each.

Two studies of high school curricula that reported an impact on separate outcomes for conceptual understanding and procedural fluency in algebra involve the *Core-Plus Mathematics Project*

[4] Developed by Carnegie Learning.

[5] Developed by the Educational Testing Service.

[6] Part of the Acuity Series developed by CTB/McGraw Hill.

(CPMP; Coxford et al., 1999), a standards-based curriculum that promotes sense making through mathematical explorations in real-world and mathematical contexts, often with the use of technology. One study (Schoen & Hirsch, 2003) found that students who completed Courses 1 and 2 (covering most of the content in Algebra I) demonstrated stronger performance on measures of understanding and ability to operate with algebra in context and did no worse on measures of algebraic procedures than students who completed a traditional Algebra I course and were matched on scores from a pretest. The second study (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000) compared the performance of students who completed Courses 1, 2, and 3 (covering all the content in Algebra I), with the performance of Algebra II students who were matched on scores from a pretest. CPMP students outperformed Algebra II students on measures of conceptual understanding and performed better (but not significantly better) on procedural items that allowed them to use a calculator (evaluating algebraic expressions, solving linear equations, etc.). However, they scored worse on items that measured pencil-and-paper symbolic manipulations without the use of a calculator. The authors suggested that "it may be that…curricula that commonly embed algebraic ideas in applied problem-solving explorations need to do a better job of helping students to abstract and articulate the underlying mathematical ideas" (p. 355) in support of pure skill development. The authors of the curriculum made this improvement in a later edition.

Research on the impact of whole-course conceptual approaches to instruction on algebra-specific student outcomes indicated that such approaches were more effective at developing conceptual understanding and, at worst, equally effective (and in some cases more effective) at developing procedural fluency than conventional approaches to teaching algebra.

Taken together, research on the impact of whole-course conceptual approaches to instruction on algebra-specific student outcomes indicated that such approaches were more effective at developing conceptual understanding and, at worst, equally effective (and in some cases more effective) at developing procedural fluency than conventional approaches to teaching algebra. However, to ensure skill development on traditional paper-and-pencil symbolic manipulations, it might be important to give explicit attention to these skills within instruction.

Implications for Program Developers and Administrators

The research reviewed here has implications for (a) using instruction that promotes procedural fluency and conceptual understanding in algebra and (b) supporting the implementation of that instruction. These are outlined in Exhibit 3. Note that each of the bullets on the right-hand side is an implication of each of the findings listed in the left-hand side.

Exhibit 3. Key Findings and Implications for Program Developers and Administrators

Algebra instruction that promotes procedural fluency and conceptual understanding...

- Engages students in activities that emphasize the meaning of algebraic symbols
- Gives students opportunities to reason about algebraic procedures by comparing solution methods
- Challenges students to struggle with algebraic concepts through prediction, investigation, and justification
- May use technology to support mathematical exploration, particularly of algebraic functions and modeling
- Engages students in modeling activities

Program developers and administrators should consider...

- Providing professional development to teachers that supports teachers in implementing these instructional practices
- Using observation protocols that target these aspects of instruction and developing ways to provide feedback on the observations guided by those protocols to teachers
- Providing teachers with curricular materials that emphasize these instructional practices
- Developing pacing guides that support the use of these instructional practices
- Providing teachers with student assessments that measure both procedural fluency and conceptual understanding

Implementation of instructional practices that promote procedural fluency and conceptual understanding will change the nature of algebra education in the United States and, potentially, have an impact on Algebra I success rates such that students who complete the course are confident and armed with strong skills and understandings of algebraic content.

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Appendix

To conduct the literature review, we followed the same process used in other briefs in this series by including descriptive, theoretical, and explanatory research on the design of curricular frameworks that are vertically aligned to support student preparation for Algebra I that spans a wide range of methodological approaches (e.g., high-quality experiments, quasi-experimental studies, descriptive studies, case studies), sources (e.g., educational journals, research organizations, national content-specific organizations), and disciplines. In addition to conducting a rigorous search of existing literature, we contacted experts in the field who are conducting research on these educational programs to identify research findings not yet published and included them in this review. We used a four-part, hierarchical selection process as the basis for including the studies summarized in this brief: subject (algebra vs. mathematics vs. other subjects), grade level (Grades 6–9 vs. Grades 1–5), year of publication (since 2005 vs. before 2005), and level of evidence (strong vs. moderate vs. low, based on standards informed by the What Works Clearinghouse; see http://ies.ed.gov/ncee/wwc/DocumentSum.aspx?sid=19). We prioritized studies that focused on algebra or mathematics in Grades 6–9, that were published since 2005, and that had strong or moderate evidence. A fully exhaustive review of the literature is beyond the scope of this brief. Instead, we focus on research studies that are most relevant for the design instructional practices that promote procedural fluency and conceptual understanding as strategies for promoting student success in Algebra I.

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