Student Success in Algebra I
Through Curricular Alignment

Perspective Brief

This perspective brief offers an in-depth look at how district math leaders and Algebra I teachers think about research on the design and implementation of curricular frameworks (i.e., a list of mathematics learning standards for each grade and course) that are vertically aligned to promote the skills and understandings needed to be successful in Algebra I in the prior grades and courses.

The Promoting Student Success in Algebra I (PSSA) project, funded by the U.S. Department of Education, recently reviewed existing research on the skills and understandings needed to be successful in Algebra I and how to sequence in prior grades and courses, but the findings from these studies may not capture practitioners' perspectives, which are shaped by their experience in the field. This brief examines whether the research findings resonate with practitioners' experience, and if not, why not. It also examines practitioners' perspectives on what program developers and administrators may need to consider when supporting the development and implementation of this strategy—the key challenges and barriers to success. Practitioners are uniquely positioned to identify key considerations given their knowledge and experience with this strategy to support struggling students.

For many students, Algebra I represents a transition from the concrete to the abstract. Rather than simply adding, subtracting, multiplying, and dividing numbers, students in Algebra I are expected to work with variables and algebraic structures. If they are not prepared to make this transition, then students might not be successful in the course. Ensuring that students begin Algebra I fully prepared with necessary skills and understandings requires a curricular framework that is vertically aligned to promote student preparation and mathematical readiness for Algebra I. Research provides recommendations for the design and implementation of such frameworks. Knowing practitioners' perspectives on this research is particularly important now, as states implement more rigorous college and career-ready standards for student learning in mathematics, including the Common Core State Standards in Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).
To better understand practitioners’ perspectives on research on the design of curricular frameworks that are vertically aligned to support student success in algebra, we asked a focus group of four district math leaders (math coordinators, coaches, and instructional leaders) and a focus group of five Algebra I teachers to read the PSSA project’s research brief outlining the evidence to date—Curricular Alignment to Support Student Success in Algebra I: Research Brief—and discuss whether and how key research findings resonated with their experience. Key findings from the research brief are summarized briefly in Exhibit 1.

Exhibit 1. Key Findings From the Review of Research on Curricular Alignment

A review of research on curricular alignment to support student preparation for Algebra I suggested that curricular frameworks (i.e., lists of mathematics standards of learning for each grade and course) should:

- Be focused and coherent.
- Emphasize the following mathematical concepts prior to Algebra I:
  - Fluency with whole numbers
  - Fluency with fractions
  - Skill in working with particular aspects of measurement and geometry
  - Algebraic thinking as an extension of arithmetic
- Be sequenced according to both the structure of mathematics and learning progressions.
- Be implemented in combination with professional development opportunities that enhance teachers’ understanding of how standards are vertically aligned across grades and courses.

Each focus group included one representative from a rural district and three or four representatives from some of the 100 largest districts across the country to ensure that the practitioners’ perspectives reflected at least some of the challenges facing both urban and rural educators. Both district math leaders and Algebra I teachers were asked to make connections between research and practice by addressing three broad questions:

- How do the research findings resonate with your experiences in the field?
- What challenges do you foresee in implementing recommendations from the research, and what supports are needed?

---

1 See http://www2.ed.gov/programs/dropout/curricularalignment092414.pdf
2 See the appendix on this brief for additional information about the methods used to collect and analyze practitioners’ perspectives.
Are there any important factors to consider that are not addressed in the existing research?

Analyses of the focus group data indicated that participants generally agreed with the findings from the research brief but provided additional thought regarding aspects of curricular alignment that have not been studied or areas of insufficient quality research, challenges faced in implementing the recommendations highlighted in the research, and supports needed to do so successfully.

In short, district math leaders and Algebra I teachers suggested that:

- Deep understanding and fluency (i.e., skill in carrying out procedures flexibly, accurately, and efficiently; NRC, 2001) with whole numbers, fractions, and integers as well as early exposure to algebraic thinking as an extension of arithmetic are important components of preparation for Algebra I.

- The standards outlined in curricular frameworks should be organized according to the structure of mathematics as well as what is known about learning progressions, and teachers need to understand how concepts are being developed across grade levels.

- Implementation of curricular frameworks that are vertically aligned to support preparation for Algebra I requires professional development to support teacher understanding as well as access to resources, including instructional materials, staffing, and funding.

These perspectives of practitioners are elaborated in the following sections, and implications of these perspectives for program development and administration are highlighted at the end of this brief. Because the perspectives highlighted in this brief represent only nine practitioners, we encourage readers to use caution when drawing conclusions. Nonetheless, these practitioners’ voices give depth and richness to the findings in the research brief.
Participants’ Perspectives on the Research

The following sections highlight the perspectives of district math leaders and Algebra I teachers that emerged from focus group discussions. These findings are organized by major themes that arose from discussions regarding the research. In each section, we begin with a brief summary of the relevant research findings followed by an exploration of key themes from the focus group participants’ reactions to the research.

KEY CONCEPTS AND ALGEBRA AS AN EXTENSION OF ARITHMETIC

Research indicates that strong preparation for Algebra I involves fluency in working with concepts typically covered in Grades K–8 mathematics, as well as an ability to think algebraically. In particular, research highlights the importance of (1) fluency with whole numbers, (2) fluency with fractions, and (3) skill in working with particular aspects of geometry and measurement (National Mathematics Advisory Panel, 2008) as well as exposure to algebraic thinking as an extension of arithmetic during the elementary and middle grades (e.g., Carraher & Schliemann, 2007). Research has also indicated that using pictures to model relationships between quantities, understanding the structural similarities between arithmetic and algebra, and using patterns to develop an understanding of functions, provides a foundation for the work done in Algebra I (Banerjee & Subramaniam, 2011; Becker & Rivera, 2007; Carraher, Martinez, & Schliemann, 2007; Carraher, Schliemann, Brizuela, & Ernest, 2006; Francsico & Hähköniemi, 2012; Freudenthal, 1974; Ng & Lee, 2009; Warren & Cooper, 2009; Warren, Cooper, & Lamb, 2006).

Focus group participants agreed that fluency with whole numbers and fractions is important to strong preparation for Algebra I. As one teacher noted, “If they could come to me with whole numbers and fractions, I could do worlds of things from there.” Both teachers and district leaders indicated, however, that fluency is not enough. Students need not only to be able to perform operations with whole numbers and fractions but also to understand the meaning behind those operations. District leaders noted that understanding of and fluency in working with integers (i.e., positive and negative numbers) should be included in the list as work with integers can facilitate work with algebraic equations.

Participants also agreed with the research findings that early exposure to algebra as an extension of arithmetic can support the transition from arithmetic to Algebra I.
Early exposure to algebra as an extension of arithmetic can support the transition from arithmetic to Algebra I.

Respondents noted that instructional approaches that promote algebraic thinking in the early grades can facilitate the development of an understanding of whole numbers, fractions, and arithmetic operations. They were particularly intrigued with the strategy of using the number line to model relationships between quantities, as demonstrated in the research brief, and its potential to promote algebraic reasoning as well as conceptual understanding. One district leader explained, “If you have an understanding of what the number is and [how to represent relationships] on the number line… I think that just translates [into] so much in algebra.” One teacher said:

I think it helps bridge the gap [between arithmetic and algebra]. Why would you subtract three from both sides of the equation $n + 3 = 15$? Why did you do that? They might say 12 is three away from 15 on the number line. So you later can tie into addition properties, properties of equality, and those kinds of things. I just think it gives them a better idea of conceptually what $n$ might be.

In addition to the use of the number line to model relationships among numbers, focus group participants discussed the importance of concentrating on the structure of numeric operations and repeated reasoning (i.e., looking for patterns) for building a foundation for algebraic thinking. One district leader explained, “You help kids learn the operations with whole numbers, with fractions based upon the structure of numbers, and then you can see that those are extendable into complex numbers, into work with polynomials.” One teacher added, “Get kids to make sense from repeated calculations and go from concrete to abstract and abstract to concrete.”

Although focus group participants immediately agreed with research suggesting that fluency with whole numbers and fractions and an ability to think algebraically as an extension of arithmetic are components of strong preparation for Algebra I, it was not immediately obvious to them that skill in geometry and measurement also are important. They needed further discussion to be convinced. One district leader said, “How could geometry… be that important? But I think it’s a lot deeper… I think it gets more into translations and transformations [i.e., movement of shapes on the coordinate plane] and a lot of those kinds of things.” One teacher explained, “The [geometry] thing caught me off guard, too, until I thought about the proportion thing.” Although they agreed with the research, participants felt that the other mathematical concepts highlighted in the research are more relevant to strong preparation for Algebra I than an understanding of geometric concepts.
SEQUENCING: STRUCTURE OF MATHEMATICS AND PROGRESSIONS

Research on student learning and curricular design indicates that mathematics content should be sequenced according to the structure of mathematics (Schmidt, Wang, & McKnight, 2005) as well as what is known about learning progressions (e.g., Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Mosher, 2011; Sarama, Clements, Wolfe, & Spitler, 2012). This research also points to the need for teachers to have opportunities to explore and understand how curriculum and standards develop across grades and courses to support student preparation for Algebra I (Center for Comprehensive School Reform and Improvement & Learning Point Associates, 2009).

Focus group participants agreed that mathematical concepts should be sequenced and vertically aligned across grade levels according to the structure of mathematics as well as what is known about learning progressions. As they talked about the sequencing of mathematics concepts, particularly the skills and understandings that the research emphasizes as necessary to be prepared for Algebra I, both district leaders and teachers noted that students do not typically enter Algebra I with those skills and understandings. This indicates that instruction at each grade level is not aligned with the curricular framework.

Teachers noted that, in particular, students do not have a deep understanding of whole numbers and fractions, nor have they been exposed to algebraic thinking as an extension of algebra. One teacher explained, “My kids don’t have that and so I end up having to reteach that before I even start to teach algebra. So my kids end up behind. If they had that, my kids would be successful.” Another teacher noted, “I think if they came in with more of the foundational work that you were talking about…we could go deeper into algebra…they don’t have that now…so we are trying to catch up.” This presents a challenge for Algebra I teachers when trying to stick to the curriculum framework and sequence outlined for their course.

District leaders suggested that, in addition to covering the material required of the grade level they are teaching, teachers feel they must preteach some of the material that will be covered in subsequent grade levels so that students are better prepared for that material. District leaders explained that because teachers feel they must preteach the following year’s math concepts, they cannot spend time fully developing the concepts in their own curriculum. As a result, students do not have an understanding of the concepts and
mathematical procedures needed to be successful in subsequent courses. For example, students can multiply fractions using an algorithm, but they may not know why that algorithm works. This impacts preparation for Algebra I.

*Teachers need to understand how a given concept develops over time, both structurally—in terms of the mathematics—and developmentally—in terms of learning progressions.*

To discourage teachers from teaching content outside of their grade level, instructional leaders and teachers agreed with the research finding indicating that teachers need to understand how the sequencing of standards in their curricular framework supports the development of concepts from year to year. Teachers can better prepare students for subsequent mathematics course, including Algebra I, if they understand how the content they are teaching builds structurally—in terms of the mathematics—and developmentally—in terms of learning progressions—to work in a subsequent grade or course.

**PROFESSIONAL DEVELOPMENT AND RESOURCES**

Research on specific supports for helping teachers understand and implement curricular frameworks that are vertically aligned to promote student preparation for Algebra I was not reviewed in the development of the research brief on curricular alignment. Yet, as they emphasized the importance of an understanding of how topics develop from year to year, district leaders and teachers noted several structures that would support successful implementation, including professional development and access to instructional, staffing, and monetary resources.

*Professional Development*

Focus group participants indicated that teachers’ instructional practice must shift when implementing vertical curricular frameworks that incorporate features important for preparing students for Algebra I. Teachers must not only teach content at a deeper level than they have in the past but, in some cases, they must also teach new and different content. To do so, teachers need professional development opportunities to examine the mathematics they are expected to teach and how that content builds for students across time.
Focus group participants suggested organizing professional development sessions around work with rich mathematical tasks. Teachers could analyze the mathematics in the tasks and then consider how, based on the standards for student learning specified in the curricular framework, students in different grade levels are expected to complete the tasks. Students in the early grades, for example, might use arithmetic to solve a word problem, whereas students in a later grade might use algebra. In conducting this analysis, teachers would discuss what happens in each grade to help students solve the problems as expected.

In addition to analysis of mathematics tasks at different grade levels, focus group participants suggested professional development activities organized around student misconceptions and student work. One district leader explained, “Bringing in student work … is very beneficial as well, having teachers really look at the student work on that progression from second, third, and fourth grade and then study that and have conversation about that.” Another district leader added, “One of the things that we’ve worked with teachers on, as part of the scope and sequence, are misconceptions and really helping teachers to understand what you are really listening for and what you pay attention to there.”

As they discussed the various professional development activities, participants emphasized that teachers should participate as part of a vertical team. One teacher explained, “I don’t think that the teachers have put two and two together. They don’t know what’s previously taught in seventh and eighth grade. They assume that they know. But they don’t really know.” Another teacher noted, “We [are] lucky to be part of a one-time vertical team meeting where you can look at some structure in progression…I think it is important to talk about similarities [across grades].”

Finally, participants suggested that professional development activities should provide opportunities for teachers to review video recordings of instruction focused on developing deep understanding of the content. As one district leader explained:

I think our teachers need to see it. They need to see someone doing it and then [they have to interact by] asking the questions … asking questions of their neighbor; why did you do that? How did you know to do that? [Engaging in that way] as learners helps them then be able to foster that in their classroom.
As teachers watch the videos and complete the activities as if they were students, they will develop a stronger understanding of the material and how to promote that understanding through their own instruction.

**Access to Resources**

In addition to access to professional development, participants emphasized the need for adequate funding, experienced staff, and administrative support in prioritizing initiatives focused on implementing curricular frameworks that are vertically aligned to support student preparation for Algebra I. Teachers and district leaders also emphasized the need for instructional resources for teachers to use in designing instruction aligned to the new frameworks, particularly resources that promote a deep understanding of concepts. These resources include documents that “unpack” the mathematics within each standard and curriculum guides that help teachers understand how to sequence topics across and within grades. Participants expressed that these resources should be developed collaboratively, with teacher input wherever possible to support teacher understanding of the learning progressions.

Adequate funding, experienced staff, and administrative support in prioritizing initiatives focused on implementing curricular frameworks that are vertically aligned to support student preparation for Algebra I are needed.

Although all participants expressed that these resources are important for successful implementation, they were particularly important to administrators and teachers in rural educational settings. Often in these districts, math coordinators must divide their time among several roles that they play. One rural district leader expressed that she was solely responsible for oversight on curricular design and assessment. As a result, teachers feel that they are on their own to find and develop resources to support their work. One rural teacher noted that teachers in these districts need access to resources from outside their districts.
SUMMARY OF PERSPECTIVES ON CURRICULAR ALIGNMENT

In summary, focus group participants provided useful insight into the design and implementation of vertically aligned curricular frameworks that promote preparation for Algebra I. They indicated that a deep understanding of and fluency working with whole numbers, fractions, and integers as well as exposure to algebraic thinking as an extension of arithmetic are important components of preparation for Algebra I. They agreed that the material should be sequenced across grade levels in a way that is consistent with the structure of mathematics and what is known about learning progressions, but they indicated that even if curricular frameworks are sequenced in this way, teachers do not always adhere to the approach. To discourage teachers from teaching outside of their grade level, it is important to help them understand how the sequencing supports development of the necessary skills and understandings across grade levels. Focus group participants suggested that professional development focused on analysis of the mathematics, student misconceptions, and student work at multiple grade levels can promote an understanding of how content develops across time. They also indicated that access to instructional, staffing, and monetary resources can support implementation of these frameworks.
Implications for Program Developers and Administrators

Findings from the analyses of the focus group data provide useful information for program developers and administrators who are working to design and implement curricular alignment. Although the practitioners often agreed with the research findings, they did expand on that research with insights relevant to their own experiences. As program developers and administrators consider the design and implementation of curricular frameworks that are vertically aligned to support student preparation for Algebra I, they should consider how the reactions of the participants in this particular project may or may not relate to their own instructional contexts. Exhibit 2 summarizes those insights and identifies potential implications relative to program implementation.

Exhibit 2. Key Findings From Focus Group Participants' Perspectives on the Research and Implications for Practice

<table>
<thead>
<tr>
<th>Key Focus Group Findings</th>
<th>Considerations for Program Developers and Administrators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students need fluency in working with and understanding whole numbers, fractions, and integers.</td>
<td>Develop curricular frameworks that emphasize skill not only in working with fractions and whole numbers but also in developing a deep understanding of these concepts. In addition, curricular frameworks should emphasize operations with integers.</td>
</tr>
<tr>
<td>Students should be exposed to algebraic thinking as an extension of arithmetic.</td>
<td>Develop curricular frameworks that emphasize algebraic thinking as an extension of arithmetic by focusing on similarities in structure, repeated reasoning, and use of the number line.</td>
</tr>
<tr>
<td>Content should be sequenced according to the structure of mathematics and learning progressions.</td>
<td>Sequence standards according to the structure of mathematics and connect outcomes to learning progressions.</td>
</tr>
<tr>
<td>Teachers need opportunities to explore the ways in which the concepts specified in a vertically aligned curricular framework build from year to year and course to course.</td>
<td>Conduct professional development activities that provide teachers with opportunities to:</td>
</tr>
<tr>
<td></td>
<td>• Analyze a given mathematics task and consider how the mathematics needed to perform that task would look different at different grades as described by the standards.</td>
</tr>
<tr>
<td></td>
<td>• Analyze student thinking across grade levels though an analysis of students’ work and misconceptions.</td>
</tr>
<tr>
<td></td>
<td>• Work with teachers of the grades or courses that come before and after the courses they teach.</td>
</tr>
<tr>
<td></td>
<td>Ensure that only the topics covered in a given grade or course are specified in the framework for that grade or course.</td>
</tr>
</tbody>
</table>
### Key Focus Group Findings

Provide teachers and district leaders with appropriate resources to support a successful implementation of curricular frameworks that are vertically aligned to support student preparation for Algebra I.

### Considerations for Program Developers and Administrators

- Provide teachers with access to instructional resources (i.e., curriculum guides and curricular materials that support instruction for deep understanding) and allocate money and time to support implementation.
- Provide district leaders with time to focus on implementation of the curricular alignment without being pulled into many directions.
References


Appendix

To examine how district policymakers and teachers use and make sense of research on the design and implementation of curricular frameworks, the project team convened a group of experienced district math leaders (including math coordinators, district math coaches, and other district-level instructional leaders) and teachers of Algebra I to participate in focus group discussions about key findings from the research.

Using a multistep process, we purposively selected focus group participants to include district administrators and teachers on the basis of their degree of knowledge and experience with the strategies of interest as well as to ensure representation of diverse types of educational contexts (e.g., rural and urban settings, middle and high schools). We identified school districts among the 100 largest local education agencies and from a list of all U.S. rural districts with which project team members had previously worked or which had been referred to us by external experts as strong candidates for discussions focused on helping struggling students succeed in Algebra I. Making sure we nominated no more than two districts from the same state, we identified 14 initial districts—10 urban and four rural—which we contacted by e-mail with information about the project and a request for an informational interview. Each nominated district was asked to nominate a teacher representative and to share biographical information for both district and teacher representatives. Nominated representatives subsequently participated in a brief interview designed to assess their experience and familiarity with five focal strategies for the Promoting Student Success in Algebra project, as well as their interest and availability in participating in the focus groups, to be conducted as part of a two-day meeting held at the offices of American Institutes for Research (AIR) in Washington, D.C. The U.S. Department of Education’s Office of Elementary and Secondary Education (OESE) and AIR ultimately selected seven of the 14 districts (five urban, two rural) from seven states located in different regions of the United States to participate in the focus groups, including one district and one teacher representative for each district (14 individuals total). District and teacher representatives collectively averaged 15 years of experience teaching math, and teacher representatives averaged nine years of experience teaching Algebra I specifically.

Participants were asked to read the research briefs in advance, and received a series of open-ended questions to guide their reading. For each of the five topic areas of focus in this study, two 90-minute focus groups with either four or five participants were conducted, one with district leaders and one with teachers. This configuration provided space for participants to focus on the issues most salient to the role they play in the district and be forthright in their responses as they were surrounded by their district- or classroom-level peers. The project team ensured that each focus group included at least one participant from a rural district. The facilitators of the focus groups were careful to
ensure that they monitored the time during the focus groups so that they covered all topics during the discussion.

The focus group protocol featured open-ended questions designed to elicit deep conversation about specific research findings from the research briefs. To facilitate conversation, each question was followed by focused probes to ascertain insights into important areas. For example, probes explored questions regarding “how,” “under what conditions,” and “why” to gain a full understanding of participants’ perspectives on each strategy as well as contextual factors that affect those perspectives.

To facilitate data collection, all focus group sessions were audio-recorded and featured a note-taker, who captured information that provided context for the audio-recording (e.g., keeping a record of which remarks came from which participant in case it was difficult to distinguish speakers on the audio-recording). Following the meeting, transcriptions of each focus group were created and content-coded. The study team analyzed and coded data with an initial set of codes based on themes that emerged in the research briefs and, in iterative fashion, codes were combined and/or revised as patterns emerged. Transcripts were double-coded and assessed for interrater agreement, with disagreements resolved to agreement. Findings from these analyses form the basis of this perspective brief, with the goal of documenting key insights from administrators and teachers on the extent to which the research resonates with their own experience and the important factors that are not addressed in the existing literature.