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# Curricular Alignment to Support Student Success in Algebra I

## Research Brief

This research brief is one of five that summarize the literature in different topic areas<sup>1</sup> related to helping struggling students in Grades 6–9 succeed in algebra. The research briefs are part of the *Promoting Student Success in Algebra I* (PSSA) project funded by the U.S. Department of Education’s High School Graduation Initiative (HSGI). The PSSA project at American Institutes for Research is designed to provide actionable information for educational program developers/administrators in three ways. First, these research briefs together will summarize research on five strategies being implemented by HSGI grantees that help struggling students succeed in Algebra I, a critical gateway course for high school graduation and enrollment in college. Second, the project includes a forum for practitioners—district curriculum developers/administrators and teachers—to make connections between the findings from the research briefs and their daily work, with the results of these discussions published in a series of perspective briefs. Third, the project includes profiles of practices that provide an in-depth look at implementation of these five strategies.

This research brief focuses on curricular alignment. Too often, students find themselves struggling to successfully complete Algebra I. One reason students struggle with Algebra I is that the course

represents a shift from working with numbers to working with variables and strings of algebraic symbols. This transition can be difficult for students, particularly if they do not feel prepared. Now, with the implementation of more rigorous College and

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<sup>1</sup> The five topic areas are Curricular Alignment, Instructional Practices, Supplementary Learning Supports, Professional Development, and Instructional Coaching.



Career Readiness Standards in mathematics and wide-scale adoption of the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010), students are being held to higher standards of learning in algebra and mathematics more generally (Kober & Rentner, 2012). Part of the challenge of raising student success rates in Algebra I is ensuring that students have the requisite skills and understandings they need to be prepared for and ultimately successful in Algebra I, particularly in this new instructional context.

What skills and understandings are needed to be prepared for Algebra I? What should districts consider as they develop and/or adopt curricular frameworks (i.e., lists of mathematics learning standards for each grade/course) that are vertically aligned to ensure that students are prepared for Algebra I? How should those frameworks be implemented? To answer these questions, we conducted a literature review. The process we used is described in the Appendix. Although much of the research addressing these questions does not meet the highest level of rigor described by the What Works Clearinghouse,<sup>2</sup> it does provide curriculum developers/administrators with factors to consider as they create and implement curricular frameworks that are vertically aligned to support student preparation for Algebra I. Our synthesis of this research suggests that curricular frameworks should (a) be focused and coherent, (b) emphasize important mathematics that is foundational to algebra, (c) be sequenced according to both the structure of mathematics and learning progressions, and (d) be implemented in combination with professional development opportunities that enhance teachers' understanding of the vertical features. These findings have implications for curriculum development, selection, and implementation and are described in the Implications section of this research brief.

## Synthesis of the Literature

### Importance of Focus and Coherence

The challenge of developing curriculum designed to support student preparation for Algebra I is one with which researchers and educators in the United States have been grappling for decades. After a series of reforms to mathematics curriculum throughout the 20th century (Klein, 2003), the National Council of Teachers of Mathematics (NCTM) released *Principles and Standards for School Mathematics* (2000), a standards document that emphasized (among other things) the importance

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<sup>2</sup> The What Works Clearinghouse was created in 2002 by the Institute of Education Sciences to be a source of information regarding what works in education. See <http://ies.ed.gov/ncee/wwc/DocumentSum.aspx?sid=19> for the standards used to evaluate studies.



of algebra with algebraic standards specified across all grade bands. This document inspired states and districts to examine their existing curricular frameworks. The result was a collection of curricular frameworks that aligned to the NCTM standards, but in varying degrees. A later report released by the Fordham Foundation found that mathematics standards for learning varied from state to state, with many providing too much emphasis on some topics and not enough on others (Klein, 2005). In addition, international comparison with other high-achieving countries indicated that mathematics curriculum in the United States was a “mile wide and an inch deep” and lacked coherence (Schmidt, Wang, & McKnight, 2005).

To provide further guidance in the development and selection of mathematics curricular frameworks, the NCTM released its *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006), which further specified the algebraic (and other mathematical) content that should be addressed at each grade level throughout Grades K–8. In addition, in 2008, the National Mathematics Advisory Panel (NMAP) released a report emphasizing that “curriculum must include (and engage with adequate depth) the most important topics underlying success in school algebra... [and be] marked by effective, logical progressions from earlier, less sophisticated topics in to later, more sophisticated ones” (p. xvii). Although the publications from the NCTM and the NMAP emphasized the need for a focused, coherent curricular framework to support student preparation for Algebra I, it was not until the release of the CCSSM (NGACBP & CCSSO, 2010) that the United States saw widespread adoption of a common curricular framework. As of September 2014, forty-three states and the District of Columbia have adopted the CCSSM and are using this framework to guide mathematics instruction. As with the NCTM standards documents, this framework is vertically aligned to support student preparation for Algebra I.

## Emphasis on Key Mathematical Topics

Curricular frameworks that are vertically aligned to support student preparation for Algebra I emphasize the skills and understandings that are necessary components of a strong preparation for Algebra I. Research in this area suggests that students should have (a) a strong background in key areas of Grades K–8 mathematics and (b) skill in algebraic reasoning as an extension of arithmetic.

## Key Concepts in Grades K–8 Mathematics

An exhaustive review of (a) curricular frameworks from high-achieving countries, (b) curricular frameworks of states that scored well in the Fordham analysis, (c) standards for student learning outlined in the NCTM’s *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A*



*Quest for Coherence* (2006), (d) survey results from a 2007 ACT survey, and (d) survey data regarding student preparation from 743 Algebra I teachers conducted by the NMAP (2008) indicates three key critical foundations of algebra: (1) fluency with whole numbers, (2) fluency with fractions, and (3) skill in working with particular aspects of geometry and measurement. Ideally, students enter Algebra I knowing their number facts, able to perform arithmetic operations, equipped with well-developed number sense, able to represent fractions in

several ways, able to convert between fractions and decimals, and able to perform fraction operations efficiently. In addition, they understand relationships between similar triangles and can solve for unknown values when working in the context of measurement. Armed with these skills and understandings, students will be ready for Algebra I.

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The topics identified by the NMAP (2008) are typically addressed in Grades K–8 mathematics. Interestingly, data collected through the teacher survey in that study indicated that student preparation in these areas is not strong (Hoffer, Venkataraman, Hedberg, & Shagle, 2007), suggesting that more must be done to develop students' understanding of the content that is found in elementary- and middle-grades mathematics. In particular, as highlighted by the NMAP standards for student learning in these areas should target not only computational skill but also conceptual understanding and problem solving. That is, students should be able to perform algorithms and have a deep understanding of the underlying mathematical ideas.

## Algebraic Reasoning as an Extension of Arithmetic

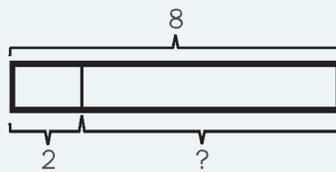
In addition to a strong background in the prerequisite skills needed for Algebra I, there is evidence that exposure to algebraic thinking as an extension of arithmetic during the elementary and middle grades supports the transition to Algebra I (Carraher & Schliemann, 2007). Recent research in this area focuses on three methods for promoting an understanding of algebraic concepts prior to Algebra I: (1) using pictures to model relationships between known and unknown quantities, (2) emphasizing similarities between arithmetic and algebra, and (3) using patterns to develop an understanding of functions.

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**Pictures to Model Relationships Between Known and Unknown Quantities.** Initial work in early exposure to algebra as an extension of arithmetic was conducted in the Soviet Union in the 1960s and 1970s. A series of experiments on different instructional approaches demonstrated that elementary-grades students could be taught to think algebraically and that when they were so taught, they tended to perform better in Algebra I (Davydov, 1969/1991). A key element of the curriculum provided to students in those studies was the use of pictures to help students find the value of unknown quantities, as shown in Exhibit 1 (Freudenthal, 1974).

### Exhibit 1. Pictures to Find Unknown Quantities

I have 2 cookies. My friend gives me additional cookies. I now have 8 cookies. How many cookies did my friend give me?



By using pictures, students developed skill in algebraic reasoning as they solved simple arithmetic problems, and this skill supported them later when they began coursework in Algebra I.

This approach to mathematics instruction is used in other countries. Our analyses of elementary-grades curricula from Singapore (a country that regularly outperforms the United States in international comparisons of student achievement) identified the use of similar models by students when solving problems involving known and unknown quantities. Research on the use of these models in Singapore found them to be valuable tools for solving problems in arithmetic and, later, algebra (Ng & Lee, 2009).

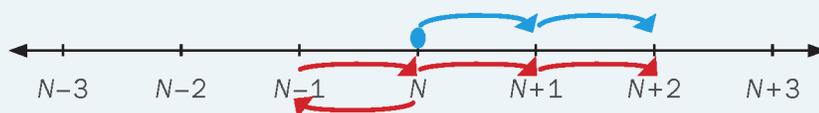
Recent research on early exposure to algebra supports the use of additional models in the elementary grades to support algebraic reasoning. Using a number line, such as that shown in Exhibit 2, to develop an understanding of relationships between known quantities can then support the transition to reasoning about unknown quantities and ultimately support an understanding of the equivalence of algebraic expressions (Carraher, Schliemann, Brizuela, & Ernest, 2006).

## Exhibit 2. A Number Line to Examine Relationships Among Quantities

The expression  $N + 2$  is two more than  $N$ .



The expression  $N + 2$  is equivalent to  $N - 1 + 3$  because they represent the same location on the  $N$ -line.



In addition, using a balance to represent equivalence of known quantities, such as that shown in Exhibit 3, can facilitate the transition to reasoning about the equivalence of expressions involving unknown quantities and, ultimately, support an understanding of methods for solving algebraic equations (Warren & Cooper, 2009).

## Exhibit 3. A Balance to Represent Relationships Among Quantities

Equivalent expressions are represented using a balance.



**Emphasis on Similarities Between Arithmetic and Algebra.** Other approaches to early exposure to algebra have focused on the structural similarities between arithmetic and algebraic expressions. Recent experimental research has indicated that a focus on the structural features of arithmetic expressions during instruction on arithmetic operations in the elementary grades supports an ability to examine the structural features of algebraic expressions, recognize the similarities in structure to arithmetic expressions, and use those similarities to simplify complex algebraic expressions (Banerjee & Subramaniam, 2011). This approach is illustrated in Exhibit 4.

## Exhibit 4. Highlighting Similarities Between Arithmetic and Algebra

In the expression  $12 + 9 + 8 \times 12$ , we treat  $8 \times 12$  as a quantity that is added to  $12 + 9$ .

Similarly, in the expression  $n + 9 + 8 \times n$ , we treat  $8 \times n$  as a quantity that is added to  $n + 9$ .

Because addition is commutative, the expression simplifies to  $9n + 9$ .

**Patterns to Develop an Understanding of Functions.** Finally, research on early exposure to algebra has indicated that elementary- and middle-grades students can understand functional relationships. Given a rule to define a linear relationship, such as the one shown in Exhibit 5, elementary-grades students were able to generate values for a functional relationship (Warren, Cooper, & Lamb, 2006) and, with support, could express that relationship algebraically (Carragher, Martinez, & Schliemann, 2007).

#### **Exhibit 5. Rule to Define a Linear Relationship**

When buying ice cream sundaes, it costs 50 cents to add sprinkles. Select three prices for ice cream sundaes without sprinkles. Give the cost of those sundaes with sprinkles. Generate an expression for the prices of a sundae with sprinkles.  
[Example expression for price of sprinkles:  $p + .50$ ]

Given geometric patterns that can be modeled by linear functions, middle-grades students were able to generate, and express algebraically, those function rules (Becker & Rivera, 2007). Middle-grades students could also identify quadratic relationships if they were first taught to generate truth sets for quadratic equations, such as that shown in Exhibit 6 (Francisco & Hähkiöniemi, 2012).

#### **Exhibit 6. Truth Set for a Quadratic Equation**

A truth set for  $y = x^2 + 2x + 1$  includes  $(-1, 0)$ ,  $(0, 1)$ ,  $(1, 4)$ ,  $(2, 9)$ ,  $(3, 16)$ ,  $(4, 25)$ , etc.

## **Sequencing According to Mathematical Structure and Learning Progressions**

The research presented in the previous sections provides evidence that curricular frameworks should focus on developing a deep understanding of the critical foundations of algebra (NMAP, 2008) as well as algebraic thinking as an extension of arithmetic. During the construction of curricular frameworks that emphasize these areas, however, questions arise as to how these topics should be sequenced across grade levels. Research on sequencing topics within mathematics curricular frameworks has not investigated the impact of one approach over another. It has, however, identified best practices.

As indicated earlier, a criticism of mathematics curricular frameworks in the United States is that the sequencing of topics is not consistent with the structure of mathematics (e.g., Schmidt et al., 2005). To evaluate the vertical alignment of curricular frameworks with respect to the structure of mathematics, one model recommends looking for (a) standards in each grade that logically connect to the preceding grades; (b) coverage of content that increases, on average, evenly in



depth across the grades; (c) coverage of content that increases, on average, evenly in breadth across the grades; and (d) new standards for each grade, such that standards for one grade are not identical to standards for another grade (Wise & Alt, 2005). These features should be evaluated for both pairs of grades and across grades (Martineau, Paek, Keene, & Hirsch, 2007).

In addition to alignment with the structure of mathematics, research suggests that standards for student learning in mathematics should be sequenced according to learning progressions. Learning progressions are empirically derived descriptions of the pathway through which student learning progresses. They are developed through an iterative process whereby researchers closely monitor the way that students attend to and organize their thinking throughout a series of instructional sequences (Mosher, 2011). Research to identify and define learning progressions within different areas of mathematics is relatively new. However, initial experimental work in pre-kindergarten has highlighted the value of instruction grounded in empirically derived learning progressions in mathematics (Clements, Sarama, Spitler, Lange, & Wolfe, 2011) and has indicated that the impact of such instruction lasts well beyond pre-kindergarten (Samara, Clements, Wolfe, & Spitler, 2012).

Guidance for how to sequence mathematics learning standards consistent both with the structure of mathematics and with what is known about learning progressions can be found in both the NCTM (1989, 2000, 2006) standards documents and the CCSSM (NGACBP & CCSSO, 2010). The NCTM standards documents provide guidance to districts and states regarding the sequencing of mathematics standards for learning across Grades pre-K–12. In addition to an emphasis on skill and deep understanding of the critical foundations of algebra identified by the NMAP (2008), the sequencing of standards in these NCTM documents promotes early exposure to algebraic thinking through the use of patterns, variables to represent unknown quantities in arithmetic expressions, and pictures to model relationships between known and unknown quantities.

Like the NCTM (1989, 2000, 2006) standards documents, the CCSSM emphasizes the development of skill and deep understanding of content associated with the areas identified by the NMAP (2008) as critical foundations of algebra as well as early exposure to algebraic thinking in mathematics coursework prior to Algebra I. The sequencing of standards within the CCSSM was informed both by the structure of mathematics and by the research on learning progressions. As a result, the CCSSM represents a set of standards that is similar to the standards of high-achieving countries in terms of coherence and focus (Schmidt & Houang, 2012) and is based on well-defined learning progressions in key areas of mathematics (Daro, 2011; Mosher 2011). This characterization of the CCSSM is true of the sequencing of standards for mathematics overall as well as for those areas identified as supporting a strong preparation for Algebra I in particular.

# Implementation Considerations: Communicate Vertical Features With Teachers

*With respect to vertical alignment, then, it is important that districts provide guidance to teachers on how to use the framework in designing instruction. It is also important that teachers understand the features of the framework that ensure vertical alignment.*

As districts prepare to implement curricular frameworks that are vertically aligned to support student preparation for Algebra I, what should they consider? Although research has not explicitly addressed the implementation of curricular frameworks designed to support student preparation for Algebra I, it has examined factors influencing the

success of mathematics reform movements. One key “take away” from this research is that the success of curricular reform is influenced by the degree to which teachers understand what is expected of students within new curricular frameworks (Cohen & Hill, 2001; Spillane, 2004).

With respect to vertical alignment, then, it is important that districts provide guidance to teachers on how to use the framework in designing instruction. It is also important that teachers understand the features of the framework that ensure vertical alignment. To do so, districts can provide pacing guides or other instructional documents that highlight the vertical progressions. In addition, districts can convene teachers to develop a vertical mapping of the content specified within the frameworks. Whatever steps are taken, teachers should understand the vertical progressions and be encouraged to work collaboratively to develop lesson plans that (a) address the big ideas of the standards and (b) build on the understandings that students are expected to develop in prior years (Center for Comprehensive School Reform and Improvement & Learning Point Associates, 2009).



# Implications for Curriculum Developers and Administrators

The research reviewed has implications for the design, selection, and implementation of curricular frameworks that support student preparation for Algebra I. These are outlined in Table 1.

**Table 1. Key Findings and Implications for Curriculum Developers and Administrators**

To support student preparation for Algebra I...	Curriculum developers and administrators should consider....
<ul style="list-style-type: none"><li>• Focus on skill and understanding of the critical foundations of algebra:<ul style="list-style-type: none"><li>▪ Whole numbers and whole number operations</li><li>▪ Fractions and fraction operations</li><li>▪ Similar triangles and solving for unknown values when working in the context of measurement</li></ul></li><li>• Expose students to algebraic concepts as an extension of arithmetic.</li><li>• Sequence standards according to the structure of mathematics and learning progressions.</li><li>• Understand the vertical features of curricular frameworks.</li></ul>	<ul style="list-style-type: none"><li>• Ensuring curricular frameworks emphasize skill and understanding of the critical foundations of algebra prior to Algebra I.</li><li>• Ensuring curricular frameworks expose students, prior to enrollment in Algebra I, to algebraic concepts as an extension of arithmetic.</li><li>• Ensuring that standards are sequenced across grade levels so that they are consistent with the structure of mathematics and what is known about learning progressions.</li><li>• Communicating the vertical nature of the standards to teachers, including mapping standards across grade bands, through professional development.</li></ul>

As curricular frameworks are implemented in districts across the country, curriculum developers, administrators, researchers, and educators will learn more about their impact and how to continue to refine them. To that end, emphasizing important mathematical concepts and procedures and sequencing standards in a manner consistent with the structure of mathematics and what is known about student learning continue to hold promise for helping students be prepared for the transition from arithmetic to algebra.

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# Appendix

To conduct the literature review, we followed the same process used in other briefs in this series by including descriptive, theoretical, and explanatory research on the design of curricular frameworks that are vertically aligned to support student preparation for Algebra I that spans a wide range of methodological approaches (e.g., high-quality experiments, quasi-experimental studies, descriptive studies, case studies), sources (e.g., educational journals, research organizations, national content-specific organizations), and disciplines. In addition to conducting a rigorous search of existing literature, we contacted experts in the field who are conducting research on these educational programs to identify research findings not yet published and included them in this review. We used a four-part, hierarchical selection process as the basis for including the studies summarized in this brief: subject (algebra vs. mathematics vs. other subjects), grade level (Grades 6–9 vs. Grades 1–5), year of publication (since 2005 vs. before 2005), and level of evidence (strong vs. moderate vs. low, based on standards informed by the What Works Clearinghouse; see <http://ies.ed.gov/ncee/wwc/DocumentSum.aspx?sid=19>). We prioritized studies that focused on algebra or mathematics in Grades 6–9, that were published since 2005, and that had strong or moderate evidence. A fully exhaustive review of the literature is beyond the scope of this brief. Instead, we focus on research studies that are most relevant for the design of curricular frameworks that are vertically aligned to support student preparation for Algebra I as strategies for promoting student success in Algebra I.



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