

## Benchmarks

The benchmarks set forth by the Panel should help to guide classroom curricula, mathematics instruction, textbook development and state assessments that will lead to proficiency with whole numbers and fractions, and competence with certain aspects of geometry and measurement.

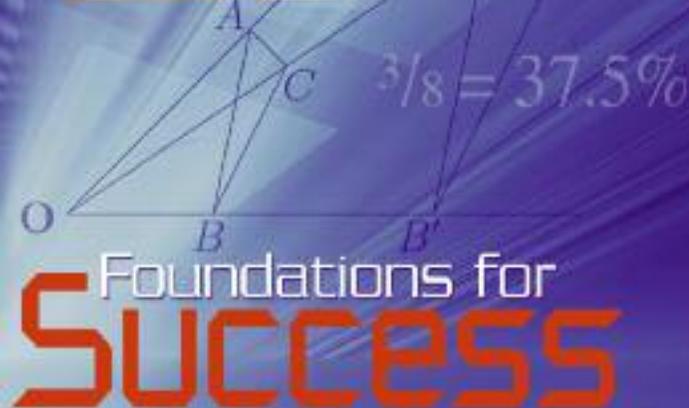
Grade Level (by the end of)	Mathematics Learning Goal
Grade 3	Proficiency with addition and subtraction of whole numbers.
Grade 4	Ability to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
Grade 5	Proficiency with multiplication and division of whole numbers. Proficiency with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals. Ability to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
Grade 6	Proficiency with multiplication and division of fractions and decimals. Proficiency with all operations involving positive and negative integers. Ability to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
Grade 7	Proficiency with all operations involving positive and negative fractions. Ability to solve problems involving percent, ratio, and rate and extend this work to proportionality. Familiarity with the relationship between similar triangles and the concept of the slope of a line.

$$x^2 - y^2 = (x-y)(x+y)$$

$$1.67 > 1\frac{2}{3}$$

$$3x - 1 = 4 + x$$

$$y \leq 10 - 2x$$



# Foundations for Success

Findings and Recommendations From the National Mathematics Advisory Panel

$$f(x) = x^2 + x - 1$$



$$x^2 = \frac{5}{2}$$

$$4 \overline{)2581}$$

$$y \geq 7x + 15$$



$$\begin{array}{r} 796 \\ + 58 \\ \hline 854 \end{array}$$

$$42 \times 13 = 546$$

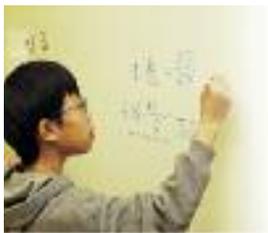
The National Mathematics Advisory Panel  
<http://www.ed.gov/MathPanel>

For electronic copies of the Math Panel's final report and the report of the task groups and subcommittees, please visit [www.ed.gov/mathpanel](http://www.ed.gov/mathpanel) or order copies at <http://edpubs.ed.gov/>, by e-mail at [edpubs@inet.ed.gov](mailto:edpubs@inet.ed.gov) and by phone at 1-877-433-7827 (TDD/TTY 1-877-576-7734).

$$3s + 1 = 4 + s$$

$$61 - 24 = 37$$

## About the National Mathematics Advisory Panel



For students to compete in the 21st-century global economy, knowledge of and proficiency in mathematics are critical. Today's high school graduates need to have solid mathematics skills—whether they are headed to college or to the

workforce. To help ensure our nation's future competitiveness and economic viability, President George W. Bush created the **National Mathematics Advisory Panel** (National Math Panel) in April 2006. The Panel was charged with reviewing the best available scientific evidence and making recommendations on improving mathematics education with a focus on readiness for and success in algebra.

The National Math Panel's final report, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, was issued on March 13, 2008. It contains 45 findings and recommendations on numerous topics, including curricular content, learning processes, instructional practices and materials, teachers, assessments and future research priorities. Highlights of the findings and recommendations are briefly summarized below. For a complete discussion of these topics, the public may read the final report as well as the reports of the task groups and subcommittees by visiting [www.ed.gov/mathpanel](http://www.ed.gov/mathpanel) or by ordering copies at <http://edpubs.ed.gov/>, by e-mail at [edpubs@inet.ed.gov](mailto:edpubs@inet.ed.gov) and by phone at 1-877-433-7827 (TDD/TTY 1-877-576-7734).

## Core Principles of Math Instruction

The areas to be studied in mathematics from pre-kindergarten through eighth grade should be **streamlined** and a well-defined set of the most **critical topics** should be emphasized in the early grades. Any approach that revisits topics year after year without bringing them to closure should be avoided. Other important findings and recommendations for mathematics instruction are:

- **Critical Foundations.** Proficiency with whole numbers, fractions, and certain aspects of geometry and measurement are the critical foundations of algebra.

- **Fractions.** Knowledge of fractions is the most important foundational skill not currently developed among American students.
- **Conceptual understanding, computational and procedural fluency, and problem-solving skills** are equally important and mutually reinforce each other. Debates regarding the relative importance of each of these components of mathematics are misguided.
- **Automaticity.** Students should develop immediate recall of arithmetic facts to free the “working memory” for solving more complex problems.
- **Major Topics of School Algebra** include Symbols and Expressions; Linear Equations; Quadratic Equations; Functions; Algebra of Polynomials; and Combinatorics and Finite Probability. More students should be prepared for and offered an authentic algebra course in Grade 8.

## Other Key Findings

- **Effort matters.** A focus on the importance of effort will improve outcomes. If children believe that their efforts to learn make them “smarter,” they show greater persistence in mathematics learning and their performance improves.
- **Most children develop considerable knowledge of mathematics before they begin kindergarten.** Children from families with low incomes and low levels of parental education and those of single parents often have less mathematical knowledge when they begin school than do children from more advantaged backgrounds. This tends to

hinder their learning for years to come. There are promising interventions to improve the mathematical knowledge of these young children before they enter kindergarten.

- **Teachers' mathematical knowledge is important for students' achievement.** The preparation of elementary and middle school teachers in mathematics should be strengthened. Teachers cannot be expected to teach what they do not know.
- **Teachers who consistently produce significant gains in students' mathematics achievement should be recognized.** When students are taught by a series of effective teachers, the positive outcomes on their mathematics learning are compounded.
- **Teachers' regular use of formative assessment improves their students' learning,** especially if teachers have additional guidance on using the assessment to individualize instruction.
- **Explicit instruction** with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation.
- **More research is needed.** The nation must continue to build the capacity for more rigorous research in mathematics education to more effectively inform policy and practice.



## Next Steps

The findings and recommendations of the Panel are a first step in the process toward fixing the delivery system in mathematics education in this country.

The United States has genuine opportunities to improve mathematics education, but these improvements can be realized only if educators, policymakers, researchers and parents all work together to ensure that our students master the mathematical foundations that are so critical for achievement in algebra and beyond.

### The National Mathematics Advisory Panel

24 expert panelists, including a number of leading mathematicians, cognitive psychologists and mathematics educators, reviewed more than 16,000 research studies before preparing a final report containing policy advice on how to improve mathematics achievement for all students in the United States.