

Math Wars: Taking Sides

Teaching students for mathematical understanding is crucial, Mr. Marshall believes. This viewpoint puts him in agreement with the thinking behind the current mathematics standards. However, he finds that the standards themselves fall short in the guidance they offer to teachers who lack the experience and confidence to teach in a way that they were not taught themselves.

BY JOHN MARSHALL

RECENTLY, I was asked which side I was on in the current math war. To clarify my thoughts, I set about re-reading some papers to remind myself what the debate was all about. *Principles and Standards for School Mathematics* was there, along with the 1989 version,¹ as were magazines, textbooks, lecture notes and handouts, lesson plans, press clippings, personal correspondence, and recollections of many conversations I have been privileged to have with some distinguished math educators around the world.

Studying the evidence in these documents has led me to the conclusion that our children are in danger

of being struck by a good deal of friendly fire. Unfortunately, they cannot escape, for they are the ones compelled to attend school. The rest of us choose to be there. Therefore it is our duty to take all the precautions we can to avoid collateral damage as the experts squabble.

Robert Reys has given us a graphic report from the war zone, where he has witnessed advocates of “reform-based” math battling “market-driven” textbook publishers.² How sad it would be if the aims of these

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two warring parties really were mutually exclusive and publishers could make money only by failing to provide our children with the mathematics they need!

Children in school today — and tomorrow — will need more mathematics than their parents did yesterday, and they will need to be taught in a far better way. More done better! That is what this debate is all about — at least I hope it is. The snag is that history shows that with change comes conflict, and that is what is happening now. The transition from an agricultural society to an industrial society was difficult. We now find ourselves in the Information Age, and we need an extensive mathematics/science base to support it. As long ago as 1968, *Your Child and Mathematics* informed parents:

Whether we like it or not, our children will be concerned in the future with more abstract mathematics than their predecessors. The world of computers and computer programs, of automatic production line processes, or of operational research by managements, is a far cry from the world of the nineteenth-century clerk, mill-hand, or small industrialist. Our most important task must be to teach children to think mathematically for themselves. From a gradual awareness of the patterns of ideas lying behind their practical experiences, there must be built up a willingness to accept the underlying mathematical ways of thinking which are proving so vital in the development of modern technological society.³

So the future has arrived. Who had a computer back then? Who doesn't have access to one now? Today, machines can do the "rote" processes for us. Thinking is the hard part. It seems to me that "we should stop training young minds to do things machines can do, but rather teach them to do things machines cannot do."⁴ Clearly, "ambitious standards are required to achieve a society that has the capability to think and reason mathematically."⁵

If thinking rules, then rote memorization has to be out, and teaching for understanding has to be in. Finding a teaching style that genuinely develops understanding is our greatest challenge, for it is not how we were taught ourselves. We should not, though, confuse committing things to memory with "teaching by rote."⁶ There are plenty of things in mathematics that need to be stored in long-term memory and recalled when required. The makeup of the human being would suggest that this task is best achieved when we understand

the information concerned. How we teach today should be dictated by what we know about how we learn.

But what has changed in mathematics education in recent times, for the world surely has? Very little, according to Michael Battista, who asserts:

For most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them. . . . Numerous scientific studies have shown that the traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem solving skills. . . . Yet traditional teaching continues, taking its toll on the nation and on individuals.⁷

Robert Reys backs up this assertion by quoting James Hiebert, who said that to assume that traditional mathematics programs have shown themselves to be successful is "ignoring the largest data base we have. . . . The evidence indicates that the traditional curriculum and instructional methods . . . are not serving our students well."⁸

In addition, an alarming piece appeared in the *Washington Post* of 21 November 2001 reporting that only about "one in five of the nation's high school seniors are proficient in math, and two in five in reading."⁹ The article continued by pointing out that "Congress raised the cap on visas granted to foreign workers to 195,000 from 115,000, largely to allow high-tech firms to fill jobs for which they could not find qualified Americans." That seemed to worry the soccer moms I know. They were anxious that their children would graduate and still not have the skills for a good job. No doubt they were aware that the Glenn Commission reported that the "technology-driven economy of the 21st century will add about 20 million jobs to the American economy by 2008 — if we can only educate our young people to fill them."¹⁰ And 2008 is not far away.

There is a huge market for the status quo that has been built up over years, so it will not be changed overnight. "No change" means that inventory is moved quickly, money is made in the short term, sales staffs are delighted, and mortgages are paid. However, those who actually pay for these profits are not so happy, as they see their children just as unsuccessful today as they themselves were yesterday. Unfortunately, the prevailing wisdom seems to be that more of the same will produce a different result. Flash cards and testing rule! To develop the war analogy further, the field hospitals are

filling up. Injections of special remediation are urgently required, which someone has to pay for. Surely prevention is better than cure? It will be cheaper in the long run, and it will also be more respectful to our children to teach them right the first time.

So the solution appears to be “test them,” and if they do not like it, tough! But do the advocates of the test-happy policies honestly believe that yet another round of testing will do the trick? American students are already the most tested students in the world.¹¹ Isn't there a danger that we spend so much time testing that we don't actually get around to teaching? Surely the evidence from the Third International Mathematics and Science Study suggests that teachers who can discuss their teaching, not testing, with knowledgeable colleagues offer children a better diet.¹² However, this ideal is easier to talk about than to achieve, for such expertise doesn't appear to be thick on the ground. Liping Ma paints a disturbing picture in her study of U.S. and Chinese teachers. She found that in the U.S. even “expert teachers, experienced teachers who were mathematically confident, and teachers who actively participated in current mathematics teaching reform did not seem to have a thorough knowledge of the mathematics taught in elementary school.”¹³ Finding a friend to phone could well be difficult.

Simplistic solutions cannot be the answer. Neither is it just a question of money, nice as that may be. It is more about changing attitudes toward teaching and learning, and changing attitudes is extremely difficult. If I were the secretary of education, the slogan on my wall would be “It's the teaching, stupid!” And I would test everyone who came to visit me to see if he or she had taken the message on board.

The evidence for change seems overwhelming, absolutely overwhelming. Children do not need more of the same, for that has surely not worked. Something drastic needs to be done. So I suppose I am tending toward the reform side in this war. But does the cure really lie in NCTM's *Principles and Standards* as they now stand? In one of the math war articles I read, the materials sponsored by the National Science Foundation (NSF) are recommended because they are based on the NCTM *Principles and Standards*.¹⁴ It seems to suggest that *only* NSF materials support the standards. But just about everything that is produced today claims to be “standards compatible” — even the pencils. Everyone I speak with has the jargon down pat. How can the classroom teacher tell the difference between the genuine article and what Walter Sawyer calls “imitation

mathematics”?¹⁵ This issue has more “spin” than my washing machine.

When attending a math conference, one is bombarded with all the “in” words. (I am still not sure what “important mathematics” is or who gets to decide.) Returning from a conference where the keynote speaker had impressed me greatly, I found a textbook that carried the speaker's name and opened it at random. The topic was square roots. I read that each chapter would have a reality orientation section, as “Real Life situations motivate ideas and provide additional settings for practice.” As it turned out, the text looked very much like the one I was taught from — apart from the absence of the tedious pencil-and-paper procedure for finding a square root of a large number, that is. (Talk about procedures that make little sense!)

As for “real life,” there were line drawings of four squares, a sketch of some Egyptian hieroglyphics, a triangle whose area is to be found from given information, and four lines on an axis, one of which could be $2x - y = 4$. In the section titled “Applying Mathematics” were four word problems of the type “Which is larger, $\sqrt{2}$ or $239 \div 169$?” Is that really “real life”?

Is this what NCTM means, I wonder, when it says, “Sometimes the changes made in the name of standards have been superficial or incomplete”?¹⁶ I doubt that the authors of the textbook would agree, for their approach seems to be consistent with this passage from *Principles and Standards*:

In grades 6-8, students frequently encounter squares and square roots when they use the Pythagorean relationship. They can use the inverse relationship to determine the approximate location of square roots between whole numbers on a number line. Figure 6.6 illustrates this reasoning for $\sqrt{27}$ and $\sqrt{99}$.¹⁷

Neither the sample text I was examining nor the two versions of the NCTM standards appear to give a *context* in which “square roots” are used. Yet all say it is their aim to see math around us. “When mathematical ideas are connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics.”¹⁸ So how useful are square roots in life? Just where are they used? There is no mention of, for example,

- The spaghetti-measuring device we have at home, which has holes for a single portion and a double portion, with the holes measuring 2.2 cm and 3.1 cm, respectively — giving an enlargement factor of 1.41 (see

Figure 1). (Recall that $\sqrt{2} = 1.414$, so $2.2 \times \sqrt{2} = 3.1$.)

- The “square” pizza boxes that we have delivered to our home, which are sized so that one holds roughly two times the amount that the other holds, with the larger measuring 14 inches and the smaller 10 inches — giving an enlargement factor of 1.4.¹⁹

- The cookie cutters we have at home, one of which has a diameter of 5.8 cm and the other a diameter of 8.2 cm, allowing one cookie to be about twice the size of the other — giving an enlargement factor of 1.41 ($8.2 \div 5.8$).

- The square cake pan we have at home, which has adjustable sides that allow one to make a cake half the size of the original square, but with the same thickness.

These things seem quite “real” to me, for I handle them frequently. Someone must have used mathematics to design them.

I find the “intelligence” in the documents I have accumulated very confusing. For example, Liping Ma’s study suggests that some U.S. teachers give an explanation of “subtraction with regrouping” that is not “a real mathematical explanation.”²⁰ She gives an example of a situation in which teachers said that “because the digit at the ones column of the minuend is smaller than that of the subtrahend, the former should borrow

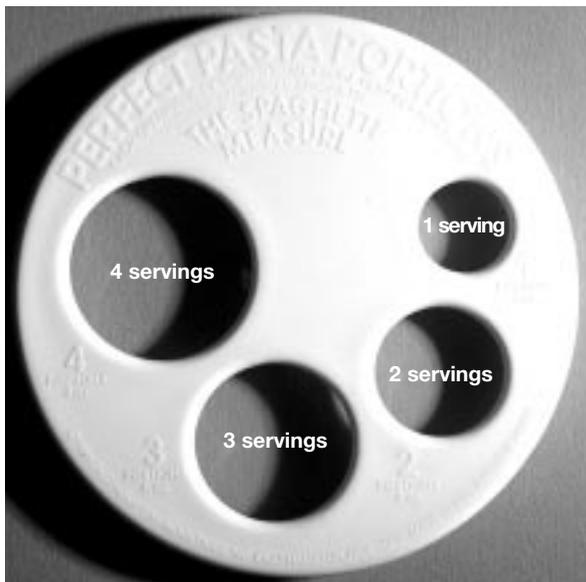
a ten from the tens column and turn it into ten ones.” Does that mean “borrowing” (interest free, of course!) is not “a real mathematical explanation”? I ask myself. Yet *Principles and Standards for School Mathematics* uses “borrowing” in its examples, and it uses “carrying” as well.²¹ Do we or don’t we “borrow”?

The answer must lie with manipulatives — doesn’t everything? But again I find it very difficult to see how using manipulatives leads us to an understanding of the efficient standard algorithms, and I cannot find much help in the documents I have. I understand that computation should be set in a context and that these contexts should be modeled with manipulatives. But how does *Principles and Standards for School Mathematics* stand up to that?

Let me turn to an example it gives in which children are discussing $728 \div 34$.²² The problem is not set in a context, although the text does say that the children are finding “the total number of 34s in 728.” That suggests to me that it is demonstrating the “measure aspect” of division. The class discussion suggests that some of these children had been taught “the old way” and that the excellent teacher is trying to bring them around to the new way. But then why, for example, does one child say, “34 goes into 72 two times,” when it is not 72 but 720 within the 728, into which the 34 goes more than two times? Is this the way we should be teaching students initially? How is this part of a curriculum for understanding? How would manipulatives be used to show this “34s in 72” technique? Is this “a real mathematical explanation” or is it “imitation math”? How does the symbolism of “long division” follow the manipulation of the manipulatives? How does the language of life match up with the language of math? It is far from clear, for I seem to have more questions than answers. If I had 728 real M&Ms to put into bags of 34 each, how would I work out how many bags I will need? Would the “manipulatives” require me to say, “34 goes into 72 two times”? In a real situation, children can “see” the math they are doing.

And then there is multiplication! If the standards advise us to model “multiplication problems with pictures, diagrams, or concrete materials”²³ so that students learn to be clear about what each number in the problem represents, why doesn’t the picture accompanying the discussion in *Principles and Standards for School Mathematics* show that?²⁴ As the student in that photo is working with his manipulatives, the posters of multiplication tables in the background look aw-

FIGURE 1.
Use of the Square Root in an Everyday Object: The Spaghetti-Measuring Device



The ratio of the diameter of the 2-serving measure to that of the 1-serving measure is not 2 — it is approximately 1.41 or $\sqrt{2}$.

fully like the “rote teaching” that has caused so much concern in the first place. What message does that give readers? Talk about a picture being worth a thousand words!

I find the suggestion in *Principles and Standards for School Mathematics* that children practice their skills by using games that require computation as part of the scorekeeping rather disappointing.²⁵ Games should be used where mathematical thinking is part of the move-generating system. In *World Class Baseball*, for example, in order to get a hit and advance around the bases, children have to make at least two numbers, rolled from three dice, match another number on a card drawn at random.²⁶ They may, for example, have to manipulate 5, 3, and 4 to match 6 (e.g., $5 + 4 - 3 = 6$). Keeping score in the game is truly insignificant when compared to the thought processes going on when young children play with numbers in this way. (In *World Class Soccer*, produced by the same manufacturers, every multiplication fact from 0×0 to 10×10 can be used to simulate the excitement of the World Cup!) Good games can offer a wide mathematical experience at different levels. Just keeping score is surely not enough.

If we now know more about how children develop, that must mean we move from concrete to abstract and back again when appropriate, with all that implies. This seems to me to be a huge issue in a subject that is about abstract ideas having real-world applications. The notion of number itself is very abstract. What many would say is at the heart of mathematics just cannot be touched or picked up. Richard Copeland addresses this in *How Children Learn Mathematics: Teaching Implications of Piaget's Research*. He argues:

Counting is often the first mathematical idea taught to children. It should not be the first mathematical activity. The idea of classification, or class inclusion, must be investigated before number can be fully understood. Classification serves as a basis, psychologically speaking, for the development of both logical and mathematical concepts. One of the most elementary types of classification problems solved by children in school is that of “sorting,” such as placing things together that belong together. Classification is fundamental to learning about the physical world. It is a basic operation in logic.²⁷

So sorting is “fundamental to learning about the physical world,” and classifying comes before counting. Yet my standards-compatible textbook starts with

counting. In counting “one, two, three, four,” how do children *understand* about “three”? I ask myself. Nor is “sorting” mentioned in the section of *Principles and Standards for School Mathematics* that deals with number. It does say, though, that children “should use empirical methods such as matching the collections, which leads to the use of more abstract methods such as counting to compare collections.”²⁸ What is meant by “leads to the use of more abstract methods such as counting”? Is something going on before counting? Just what does it mean to “understand” number? What is going on in the classroom to support developing an understanding of number? What do the children play with? What do they do? What is displayed on the walls? How is it different from what used to be? It doesn't seem very clear to me, and I think it should be.

I keep asking myself over and over whether there is some paradox in all of this. Are the standards themselves “superficial or incomplete,” as they describe some texts?²⁹ I am beginning to wonder — at least with regard to the foundation-laying elementary school. If standards-based materials “help students make sense of mathematics in several ways,” as Trafton, Reys, and Wasman claim,³⁰ am I reading the wrong material? But I am reading the exemplars included in the standards documents themselves, where, for example, I am advised that it is “misleading” to speak of an “upside-down triangle,”³¹ yet I am encouraged to talk about “thick and thin” ones.³² I also learned there that “the test” accepted that a (spherical?) balloon was — like a soccer ball and “my sister's bra” — a circle. No. No. No! As I said before, it's the teaching, stupid!

Over the years I have spent considerable time discussing with some talented math educators just what we mean, and do, when we teach multiplication for understanding. I have further explored this question by working with some delightful children. Let me now offer an alternative version of how we might approach this “hot” topic. It is a suggestion to be debated and improved upon, and should not be regarded as a set of rules. Equally, the set of rules that form the basis of mathematics should not be regarded as suggestions. I hope my comments add to the deliberations about a problem that really requires a design solution and not just some tinkering at the edges.

Before I begin, let me review what *Principles and Standards for School Mathematics* advises on page 151.

It is important that students understand what each number in a multiplication and division expression

represents. For example, in multiplication, unlike addition, the factors in the problem can refer to different units. If students are solving the problem 29×4 to find out how many legs are on 29 cats, 29 is the number of cats (or number of groups), and 4 is the number of legs on each cat (or the number of items in each group), and 116 is the total number of legs on all the cats. Modeling multiplication problems with pictures, diagrams, or concrete materials, students learn to be clear about what each number in the problem represents.

Against this background, here are my comments. Much has been made of the need to move away from the tradition of teaching by “rote memorization” to one of teaching for understanding. We have made a powerful argument for this approach, the major point being that the human brain functions best when it “understands.” We have also made the case for how this should be done — with “concrete models,” that is, real situations.

Moving the discussion specifically to the teaching of multiplication in elementary school, we find that the way the subject has been taught in the past makes teaching for understanding difficult. In the past we have drilled students to read sentences such as $3 \times 4 = 12$ as “three 4s are 12.” From this has come the common shorthand, “three times four is 12.” But the topic is multiplication, and 3×4 means $3 + 3 + 3 + 3$, and *not* $4 + 4 + 4$. Euclid wrote about this over 2,000 years ago, saying, “One number is said to multiply another when the number multiplied is so often added to itself, as there are units in the [second] number, and another number is produced.” More current dictionaries (Webster’s Third, College Edition) say: “Multiplication: the process of finding the number or quantity (product) obtained by repeated additions of a specified number or quantity (multiplicand) a specified number of times (multiplier); symbolized in various ways (ex. $3 \times 4 = 12$ or $3 \cdot 4 = 12$, which means $3 + 3 + 3 + 3 = 12$, to add the number three together four times).”

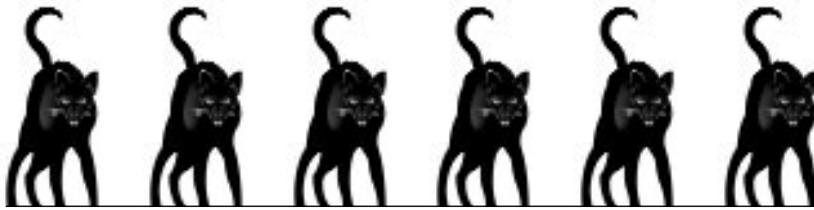
In a curriculum that advocates understanding the operations, children need to feel comfortable knowing that 3×4 means $3 + 3 + 3 + 3$ and equals 12 and that it may come from a variety of situations. Experience has shown that students who come to the idea without preconceptions feel more “at home” with this concept than do teachers who are steeped in a “rote memorization” tradition. These teachers often feel rather intimidated and somewhat exposed finding that some

mathematics they felt was secure is challenged. Some are even annoyed with their past. (Unlearning is so much harder than learning properly the first time around!) Indeed, many of these teachers argue that 3×4 *means the same* as 4×3 because $3 \times 4 = 4 \times 3$! But this is not so. The *meaning* of multiplication is not commutative, even though the *operation* of multiplication is. The two should not be confused but should form part of what Liping Ma calls the “knowledge package” for multiplication, which in turn is part of the “profound understanding of fundamental mathematics” that elementary school teachers need to develop.³³

The difficulty many adults have in accepting that meanings are not commutative demonstrates how hard it is to overcome a “rote memorization” background in favor of teaching for understanding. Young, uncluttered minds do not have this problem. As we have repeatedly stressed, it is important that students understand what each number in a number sentence represents, and this applies to both multiplication and division. For example, in multiplication, unlike addition, the factors in the problem can refer to different units. If students are solving the problem to find out how many legs there are on six cats, the model will show $4 + 4 + 4 + 4 + 4 + 4$ (see Figure 2). That is, a set of 4 (legs), repeated 6 times. The accompanying number sentence associated with this problem is 4×6 , where 4 is the number of legs on each cat (the multiplicand) and 6 is the number of cats (the multiplier), with 24 as the total number of legs on the cats (the product). “Modeling multiplication problems with pictures, diagrams, or concrete materials, students learn to be clear about what each number in the problem represents.” Using concrete materials, or representations of concrete materials, will make it clear to young children that 4×6 cannot, for example, represent six cats each with four legs, while at the same time representing four cats with six legs each! They quite rightly do not believe these are “the same.” The overriding need to work at the concrete operational stage of a child’s development, the necessity of offering young learners clear images of mathematical concepts, and our own “clash with the past” are all parts of the challenge of teaching for understanding.

David Johnson and Julia Anghileri recognize that difficulties may arise when introducing children to the “x” sign and *suggest* that it be left until later: “It is not wise to introduce the multiplication symbol, x, at this stage.”³⁴ They *suggest* that the symbolism $3(2) = 6$ be used to denote situations that come from $2 + 2 + 2$ as a *prelude* to the introduction of the “x” sign. Here,

FIGURE 2.
Modeling a Multiplication Problem with a Picture



$$4 + 4 + 4 + 4 + 4 + 4$$

$$4 \times 6 = 24$$

(4 legs, 6 times → 24 legs)

3(2) is read as “3 sets of 2.” Later, 2×3 will be read as “2 multiplied by 3” and come from contexts that mean $2 + 2 + 2$. There is much to commend this approach, which is illustrated in Figure 3.

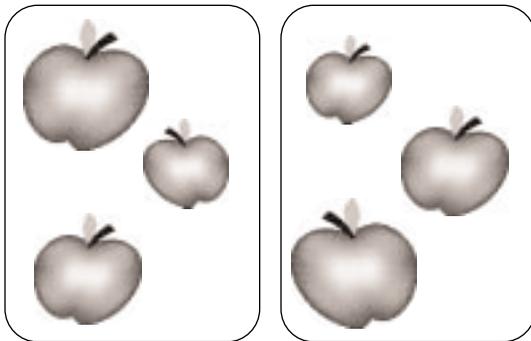
Developing an “at homeness” with small numbers allows confidence to grow in young children. There is nothing to be gained by rushing to handle big numbers before the first stage is secure. A curriculum that supports understanding will stress the *language* of num-

ber sentences, such as 2×3 , and will therefore spend considerable time, in a variety of ways, dealing with products in the range up to 5×5 . The point is that knowing the *meaning* of 2×3 will help children understand the solution to unknown products, such as 6×7 . Knowing that the *solution* to 2×3 is 6 does not help find the answer to 6×7 . We must concentrate on the processes of mathematics instead of just the results of mathematics. Children need to develop these “power skills.”

FIGURE 3.
A Preferred Way to Write a Number Sentence

Kirsten sorts 6 apples in 2 different ways.

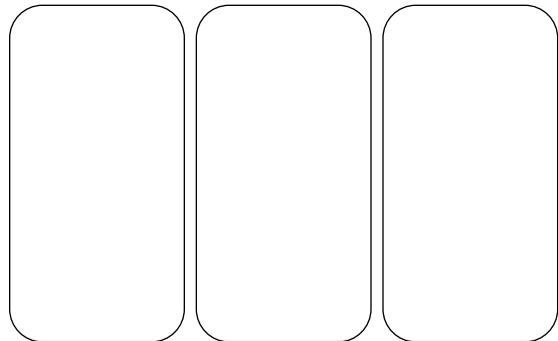
First she makes 2 sets of 3.



She has a set of
 3 apples 2 times.
 $2(3)$ can be written 3×2
 $3 \times 2 = 6$

Kirsten says: 3 multiplied by 2 equals 6.

Then she makes 3 sets of 2. Complete.



She now has a set of
 2 apples ___ times.
 $3(\underline{\quad})$ can be written $\underline{\quad} \times 3$
 $\underline{\quad} \times 3 = 6$

Kirsten says: _____ equals 6.

Adapted from Sir Wilfred Cockcroft, John Marshall, et al., *New Curriculum Mathematics for Schools* (Harlow, Essex, U.K.: Oliver and Boyd, 1989). Used by permission of the author.

Many teachers and parents will quite rightly want their children to commit number facts to memory. Indeed, it is so crucial to future performance that it is too important to “mess up.” As Sir Wilfred Cockcroft put it in a major report on the teaching of mathematics in England and Wales:

Well-mastered routines are necessary in order to free conscious attention as much as possible so that it can focus on aspects of a task which are novel and problematic. Here again, we need to distinguish between “fluent” performance and “mechanical” performance. Fluent performance is based on understanding of the routine that is being carried out; mechanical performance is performance by rote in which the necessary understanding is not present. Although mechanical performance may be successful in the short term, any routine which is carried out in this way is much less likely to be capable of use in other situations or to be retained in long term memory.³⁵

Mathematics needs to be used. Cramming for the test may well give false results if the mathematics assessed is allowed to atrophy and cannot be used later. Feeling “at home” with multiplication facts is not a fad. Feeling “at home” with mathematics is every child’s right. To be genuinely good at math is a wonderful goal.

So where does this leave me? I find myself in complete agreement about the need for reform. Let there be no doubt about that. We cannot go on killing as many young minds as we have done for generations, for they will grow up and replicate the problems we now have, and nothing will have changed. And yes, the aims and aspirations of current NCTM standards are to be commended. I just don’t see how many of the exemplars offered match those aims. To me, they simply do not seem to reflect the attitude toward the teaching/learning process that the aims talk about, and thus they are surely not “what children need.”

The debate in schools must now be about teaching mathematics. Mathematically able and enthusiastic leaders must be developed — and suitably rewarded — to lead this debate. It is not a job for just anyone. The Catch-22 is that, because of the way they were taught, too many teachers and prospective teachers lack any mathematical confidence themselves. However, there are a number of first-rate, young-at-heart personalities out there who, with a different leadership, are just waiting

to attack the future. It is they who must be the solution, and they must be nurtured. With guidelines whose exemplars truly match the stated aims, together with day-to-day support in which they have confidence, these teachers will be able to inspire their students and lay a firm mathematical foundation on which others can build. In time, they will become the new leaders. When they do, I hope they will never forget that they came from the classroom. I hope they never forget what mathematics teaching is all about, for it is not easy to learn or indeed to teach. I hope, too, that they do not create a new bureaucracy to replace the one they surely will have dismantled. And when they become keynote speakers, I hope I am there to hear them.

So whose side am I on in this math war? Why, the children’s side, of course!

1. *Principles and Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 2000); *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: NCTM, 1989).
2. Robert Reys, “Curricular Controversy in the Math Wars: A Battle Without Winners,” *Phi Delta Kappan*, November 2001, pp. 255-58.
3. Wilfred H. Cockcroft, *Your Child and Mathematics* (Edinburgh: Chambers and Murray, 1968), p. 2.
4. John Marshall, comments to Minnesota Mathematics Framework Review Conference, St. Paul, 1997; see also *1997 Minnesota K-12 Mathematics Framework* (St. Paul: Minnesota Department of Children, Families, and Learning, 1998), Number Sense, p. 6. Available on the Web at www.scimathmn.org/frameworks_math.htm.
5. *Principles and Standards for School Mathematics*, p. 29.
6. The *Oxford English Dictionary* defines “by rote” as “in a mechanical manner, by routine; especially by the mere exercise of memory without proper understanding of, or reflection upon, the matter in question.”
7. Michael T. Battista, “The Mathematical Miseducation of America’s Youth: Ignoring Research and Scientific Study in Education,” *Phi Delta Kappan*, February 1999, p. 426. See also Michael T. Battista and Carol N. Larson, “The Role of JRME in Advancing Learning and Teaching of Elementary School Mathematics,” *Teaching Children Mathematics*, November 1994, pp. 178-82; and Mary Montgomery Lindquist, *Results from the Fourth Mathematical Assessment of the National Assessment of Educational Progress* (Reston, Va.: National Council of Teachers of Mathematics, 1989).
8. James Hiebert, quoted in Reys, p. 258.
9. Michael A. Fletcher, “Students Found Lacking in Science,” *Washington Post*, 21 November 2001.
10. *Before It’s Too Late: A Report to the Nation from the National Commission on Mathematics and Science Teaching for the 21st Century* (Washington, D.C.: National Commission on Mathematics and Science Teaching for the 21st Century, 2000).
11. John Morrow, “American Elementary and Secondary School Students Are Tested Far More Than Their Counterparts in Other Industrial Nations,” *Education Life*, special supplement, *New York Times*, 13 January 2002.
12. James W. Stigler, “Briefing for the National Commission on Mathematics and Science Teaching for the 21st Century,” Washington, D.C., 23 September 1999, pp. 19, 23 of the transcript.

(Continued on page 249)

15. Walter W. Sawyer, *Mathematician's Delight* (Baltimore: Penguin Books, 1943), p. 8.
16. *Principles and Standards for School Mathematics*, pp. 5-6.
17. *Ibid.*, p. 220.
18. *Curriculum and Evaluation Standards for School Mathematics*, p. 32.
19. Readers may care to note that, to be more precise, for the smaller box to be half the volume of the larger box, the length of the side would need to be $\sqrt[3]{98}$ inches, i.e., 9.90 inches. The school text mentioned above had children deal with $\sqrt{99}$ — on a number line!
20. Ma, p. 22.
21. *Principles and Standards for School Mathematics*, pp. 140, 85.
22. *Ibid.*, p. 153.
23. *Ibid.*, p. 151.
24. *Ibid.*, p. 142.
25. *Ibid.*, p. 87.
26. World Class Baseball is produced by World*Class Learning Materials, Inc., 111 Kane Street, Baltimore, MD 21224.
27. Richard W. Copeland, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: Macmillan, 1970), p. 32.
28. *Principles and Standards for School Mathematics*, p. 122.
29. *Ibid.*, p. 6.
30. Trafton, Reys, and Wasman, p. 261.
31. *Curriculum and Evaluation Standards for School Mathematics*, p. 49.
32. *Ibid.*, p. 30.
33. For “knowledge package,” see Ma, pp. 76-78; and for “profound understanding of fundamental mathematics,” see Ma, pp. 120-23.
34. David Johnson and Julia Anghileri, “Arithmetic Operations on Whole Numbers: Multiplication and Division,” in Thomas R. Post, ed., *Teaching Mathematics in Grades K-8: Research Based Methods* (Boston: Allyn and Bacon, 1988), p. 159.
35. Committee of Inquiry into the Teaching of Mathematics, *Mathematics Counts* (London: H.M.S.O., 1982), p. 70. 

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(Continued from page 200)

13. Liping Ma, *Knowing and Teaching Elementary Mathematics* (Mahwah, N.J.: Erlbaum, 1999), p. xix.
14. Paul R. Trafton, Barbara J. Reys, and Deanna G. Wasman, “Standards-Based Mathematics Curriculum Materials: A Phrase in Search of a Definition,” *Phi Delta Kappan*, November 2001, pp. 259-64.

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